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ECONOMIC OPERATION OF A HYDRO-THERMAL SYSTEM CONTAINING COMMON-FLOW HYDRO-ELECTRIC PLANTS

by

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UNIVERSITY OF ALBERTA

FACULTY OF GRADUATE STUDIES

The undersigned certify that they have read, and recommend to the Faculty of Graduate Studies for acceptance, a thesis entitled "Economic Operation of a Hydro-Thermal System Containing Common-Flow Hydro-Electric Plants" submitted by Rajinder Kumar Verma in partial fulfilment of the requirements for the degree of Master of Science.

ABSTRACT

This thesis investigates the problem of economic scheduling of an interconnected hydro-thermal power system, over short intervals. The minimization of the operating cost of the system is treated as a Lagrange problem using the Calculus of Variations. A new method has been developed for obtaining scheduling equations for the hydro-plants of the system. The effect of the variation of head upon the hydro-plant characteristics and the transmission line losses are taken into consideration.

The scheduling equations for the hydro-plants located on separate streams are shown to be equivalent to those derived by Glimn and Kirchmayer [14].

New scheduling equations are developed in this thesis for the common-flow hydro-plants of the system. For simplicity it has been assumed that only two hydro-plants are located on the same stream. The time taken by water to flow from the upstream plant to the downstream plant is taken into account. The method is general in nature and can be extended to cases where more than two hydro-plants are located on the same stream.

The scheduling equations are applied to a simple power system to test their effectiveness. The computing technique used for doing so is also discussed.

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A_j	Surface area of the reservoir of the j^{th} hydro-plant, in Acres.
$C_{T_i}(P_{T_i})$	Cost function for the i^{th} thermal plant, to represent the cost to generate P_{T_i} for one hour, in \$/hr.
$F_j(t)$	Natural inflow to the reservoir of the j^{th} hydro-plant, in K.S.F. (1 K.S.F. = $1x10^3$ ft^3/sec).
h_j	Net head at the j^{th} hydro-plant, in ft.
h_{Fj}	Forebay elevation at the j^{th} hydro-plant, in ft.
h_{Tj}	Tailrace elevation at the j^{th} hydro-plant, in ft.
h_{Lj}	Loss of head due to the friction in conduits of the j^{th} plant, in ft.
P_D	Total load demand of the system, in Mw.
P_{Hj}	Power output of the j^{th} hydro-plant, in Mw.
P_L	Transmission line losses, in Mw
P_{Ti}	Power output of the i^{th} thermal plant, in Mw.
q_j	Discharge from j^{th} hydro-plant, in K.S.F.
Q_j	Total amount of water to be used by the j^{th} hydro-plant, during the optimization interval, in (1 K.S.F.- hr = $3600x10^3$ ft^3).
s_j	Storage at the j^{th} hydro-plant, in K.S.F. - hr.
$\dot{s}_j$	Rate of change of storage at the j^{th} hydro-plant, in K.S.F.

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T	Optimization interval, in hours.
γ_{j0}	Water conversion coefficient for the j^{th} hydro-plant at $t=0$, in \$/K.S.F.-hr.
λ	Lagrange multiplier, used as the incremental cost of the system, in \$/Mw-hr
σ_j	spillage at the j^{th} hydro-plant, in K.S.F.

CHAPTER I

INTRODUCTION

1-1 General

The generation of electrical power by thermal and hydraulic means, using the natural resources such as water and coal, represents a very important factor in a nation's economy. The effective utilization of these natural resources, for the generation of electrical power, will reduce not only the operating costs, but also future capital commitments for additional generating equipment. Relatively small deviations from the optimal coordination of the expenditure of these resources can cause considerable monetary losses over a period of time.

Thus, the problem is basically one, in which the limited available natural resources have to be most effectively utilized to meet certain load requirements. The generation schedules for each of the thermal and hydroelectric plants should be such that the cost of operating the system, over the given optimization period, is a minimum.

The methods of scheduling the operation of pure thermal systems [1,2,3], by using the incremental cost techniques, have been widely discussed in the literature and have been put to extensive use. The problem of preparing an economic operating schedule for a hydrothermal system is more complex. The essential difference

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between the operation of an all thermal and a hydro-thermal system is that the operation of the thermal plants depends only upon the conditions which exist from instant to instant, while the operation of the hydrothermal system depends upon the conditions which exist over the entire optimization interval [4].

The hydro-thermal systems can be divided into two catagories. In the first, the hydraulic plants are located on separate streams, and in the second some or all the hydraulic plants are situated on the same stream. In the latter case, the discharges from the upstream plants affects the operation of the downstream plants as well [5]. In this case, the time taken by the water to flow from the upstream plants to the downstream plants should also be taken into consideration.

The optimization problem can be classified on the basis of the duration of the optimization period:

(a) Long-range problem - The period of optimization may range from a few months to one year.

(b) Short-range problem - The optimization interval may extend from 24 hours to a week.

The difficulties in predicting the natural inflows, makes the long range problem rather complex. As a matter of fact, only probabilistic techniques may give worthwhile results for a long-range problem. It is relatively easier

to predict the natural inflows over shorter intervals. A technique similar to the Basic Rule Curve [6], may be used to decide the amount of water, which is to be used over the optimization interval of 24 hours. The short-range problem is the subject of investigation in this thesis.

The short-range problem has been attacked by other investigators in several ways. In almost, all cases a mathematical model of the system is constructed and solved. The method of formulating the problem depends upon the mathematical technique adopted for solving it. The Variational Calculus [7,8,9], the Maximum Principle [9], Dynamic Programming [9] and the Gradient Methods [9] have been successfully used. Dynamic Programming and the Gradient Methods, for a given set of conditions, actually compute all possible combinations of the thermal and hydro-plant generations from hour to hour; and then select that combination which results in minimum cost over the optimization interval.

The methods of Calculus of Variations, and the Maximum Principle depend upon the iterative calculations which converge to the desired answers. The feasibility of using Pontryagin's Maximum Principle for preparing the optimal generation schedules, has been shown by Dhalin and Shen [10], and Hano, Tamure and Narita [11]. But as the number of the plants in the system increases,

it becomes quite difficult to match the two point boundary conditions for the state and the adjoint equations and to evaluate the Hamiltonian numerically.

The Variational techniques have been extensively discussed and used to obtain the optimal generation schedules. Ricard [12] used the Variational Calculus to arrive at the well-known Ricard's equation. Cypser [5] posed the problem using the Calculus of Variations and solved the resulting Euler equations by the Steepest Descent method. He discused the common-flow problem as well. Chandler, Dandeno, Glimn and Kirchmayer [13] advanced the concept of incremental cost to combined hydro-thermal systems, by treating the hydro-plants as constant head plants. Glimn and Kirchmayer [14] improved upon this method by considering the effect of variation of head upon the hydro-plant characteristics. Arismunandar [1] made an effort to formulate the problem for a combined hydro-thermal electric system, but made no effort to consider systems containing the Common-flow plants. Menon [15] used the Calculus of Variations approach to formulate the problem and then used the Euler equations for constructing sets of minimizing sequences.

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1-2 Scope of the Thesis

The purpose of this investigation is to obtain, using Calculus of Variations, general scheduling equations for a hydro-thermal power system containing common-flow hydro-electric plants. The integral of the cost of operating the thermal plants of the system, over the optimization interval, is taken as the performance index for the problem. The transmission losses in the system and the effect of variation of head upon the hydro-plant characteristics are taken into consideration.

The scheduling equations for the hydro-plants located on separate streams, are shown to be equivalent to those by Glimn and Kirchmayer [14] (their equations are extensions of Ricard's equations). New coordinating equations are obtained for plants located on the same stream.

An example to illustrate this method is discussed. The system, under study, consists of one thermal and two hydro-plants. The hydro-plants have storage facilities on the same river. The transmission losses and the time lag between the reservoirs are considered negligible. The scheduling equations for the common-flow plants are simplified for the given system.

The numerical results are obtained with the help of the IBM-360 digital computer system. Fortran IV programming language is used. The programme is discussed in some detail.

CHAPTER II

SYSTEM CHARACTERISTICS AND CONSTRAINTS

2-1 A General Statement of the Problem

Consider a system containing M thermal and N hydro-electric plants, interconnected electrically by a network of transmission lines. A predetermined limited amount of water is to be used by each hydro-plant, over the optimization period. The natural inflows and the load demand are known functions of time. It is desired to operate this system to meet the load demand in such a way that the total operating cost of the system, over the optimization interval, is a minimum. Some or all the hydro-plants may be located on the same stream. The transmission losses and the effect of variation of head upon the hydro-plant characteristics are to be taken into consideration.

In order to construct a suitable mathematical model of the system, the characteristics of the various elements of the problem should be examined in detail.

The main elements of the problem are:

(i) Hydro-electric power plants,

(ii) Thermal plants and the cost function,

(iii) Transmission losses in the system, and

(iv) The system restrictions.

2-1 Hydro-electric Power Plants

The presence of the hydraulic plants in a system makes the problem quite complex. Inaccuracies in predicting the natural inflows to the reservoirs, further complicates the problem. In the following analysis it is assumed that the natural inflows to the reservoirs of the hydro-electric plants are known functions of time.

It is, however, interesting to compare the performance of the steam and hydraulic power plants. A hydraulic turbine can go from a stationary position to full load in 1-3 minutes [16], while it may take hours to fire up a steam plant and put it on line. A hydraulic turbine spinning under no load will pick up load almost instantaneously. The hydraulic plants are inexpensive means of reliable reserves and are quite adaptive to load variations.

The output of a hydraulic turbine is given by:

$$P_H = \frac{q.h.\eta}{11.8} \quad \text{Kw} \tag{2.1}$$

where

P_H = Output of the turbine in Kw

q = discharge in ft^3/sec

h = net head in ft

η = efficiency of the turbine, under the operating conditions.

The efficiency (η) of a turbine depends upon its design and the operating condition. Thus, for a given turbine, η can be represented as a nonlinear function of the discharge and the net head. A hydro-electric power plant consists of a number of such turbines. In general, the output of a power plant can be represented as a function of the head and the discharge. This is done by collecting the input-output data for a plant and fitting surfaces into this experimental data [14]. Thus, the output of the j^{th} hydroplant (at any instant of time t) can be written as:

$$P_{Hj} = P_{Hj}\ (q_j,\ h_j,\ t) \qquad (2.2)$$

The problem stated in section 2-1, is posed as a fixed end point Lagrange problem in the Calculus of Variations in the subsequent chapters. To do this, it is desirable to represent the output of the hydro-electric plants as a function of storage at the plant and the rate of change of storage at the plant and at all the upstream plants (if any) [5]. This is done by representing the discharge and the net head at the plant as functions of the storages and their time differentials as explained in the following paragraphs.

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In the following analysis, it is assumed that only the k^{th} and $k+1^{st}$ plants are located on the same stream (both plants have adequate reservoir capacity), the rest of the plants are located on separate streams.

(a) Discharge

(i) Plants on separate rivers: The discharge can be represented as a function of the natural inflow to the reservoir of the plant and time differential of the storage at the plant. Thus for the j^{th} plant

$$q_j(t) = F_j(t) - \dot{s}_j(t) \tag{2.3}$$

(ii) Common-flow plants: When the hydroplants are located on the same stream, the discharges of the upstream plants affect the performance of the downstream plants as well. The water released from the upstream (k^{th} hydro-plant)plant takes time τ to reach the downstream plant ($k+1^{st}$ hydroplant); that is, the water released at an instant $(t-\tau)$ from the k^{th} plant shall reach the k+1st hydroplant at an instant t. Hence, for the k^{th} plant

$$q_k(t) = F_k(t) - \dot{s}_k(t) \tag{2.4}$$

and for the $k+1^{st}$ plant

$$q_{k+1}(t) = F_{k+1}(t) + q_k(t-\tau) - \dot{s}_{k+1}(t) \qquad (2.5)$$

The above expressions can easily be extended to a case in which more than two plants are located on the same stream.

(b) Net head

The available head for power generation is given by the difference between the forebay and the tailrace elevation minus the losses due to the friction, etc. in the conduits. That is, net head at the j^{th} hydroplant

$$h_j = h_{Fj} - h_{Tj} - h_{Lj} \qquad (2.6)$$

The forebay elevation can be represented as a function of storage in the reservoir of the plant while the tailrace elevation and the frictional losses can be represented as functions of discharge. The discharge, however, can be represented as a function of time differentials of the storages as discussed in the previous paragraphs. Thus the net head can be represented as a function of storage at the plant and of the time differential of the storage at the plant and at the upstream plants (if any).

(c) Output of the Hydro-electric Plants

The output of a hydroplant is a function of discharge and net head at the plant, as discussed while deriving the equation (2.2). The discharge and the net head can be represented as a function of the storage and the time differential of the storages and the natural inflows. As the natural inflows are uncontrollable variables, and since their values at various instants are known, the plant outputs can be represented as functions of the storages and their time differentials only.

(i) Plants on separate rivers:

$$P_{Hj} = P_{Hj}\,(s_j,\ \dot{s}_j,\ t) \tag{2.7}$$

(ii) For the common-flow plants:

The output of the k^{th} (upstream) plant is given by:

$$P_{Hk} = P_{Hk}\,(s_k,\ \dot{s}_k,\ t) \tag{2.8}$$

and, the output of the $k+1^{st}$ (downstream) plant is given by,

$$P_{Hk+1}(t) = P_{Hk+1}(s_{k+1}(t),\ (q_k(t-\tau)-\dot{s}_{k+1}(t)))$$

$$= P_{Hk+1}(s_{k+1}(t),\ (\dot{s}_k(t-\tau)+\dot{s}_{k+1}(t))) \tag{2.9}$$

$\dot{s}_k(t-\tau)$ can be represented as some nonlinear function of $\dot{s}_k(t)$ and τ, i.e.,

$$\dot{s}_k(t-\tau) = f(\dot{s}_k(t), \tau) \qquad (2.10)$$

If τ is considered to be a constant, then,

$$P_{Hk+1}(t) = P_{Hk+1}(s_{k+1}(t), f(\dot{s}_k(t)), \dot{s}_{k+1}(t))$$

$$= P_{Hk+1}(s_{k+1}, \dot{s}_k, \dot{s}_{k+1}, t) \qquad (2.11)$$

2-3 Thermal Plants and the Cost Functions

The problem,under study, is one in which the cost of operating the system over a given interval is to be minimized, while it is desired to meet a given load demand. The operating costs are only those costs which are directly related to the output of the generating units. The capital costs on the reservoirs and the generating units are not taken into account here, since these are not functions of the power outputs. Thus the cost of operating the system is attributed to the cost of operating the thermal plants only. The operating cost [17] of a thermal plant consists of:

(i) Starting cost,

(ii) No load spinning cost,

(iii) The loading cost.

The starting cost depends upon the down-time of the units. If a unit is stopped and then restarted immediately, there will be no or little starting cost. In general,

$$\$_{starting} = \$_0 \; (1-e^{-at}) \tag{2.12}$$

where

a = cooling rate

$\$_0$ = cold start cost

The no load and the loading cost indicate the fuel expenditure to run the unit once it has been started and placed on line. The no load cost is a constant rate of fuel expenditure incurred as long as the unit is running, while the loading costs are directly related to the output of the thermal plants.

By making suitable assumptions, the cost of operation of a thermal plant can be represented as a quadratic or cubic polynomial of the output of the plant.

Thus, for the i^{th} thermal plant,

$$C_{Ti} = C_{Ti}(P_{Ti}, t) \tag{2.13}$$

$$i = 1, 2, \ldots., M$$

The cost function for the whole of the system can be represented as a sum of the cost functions of all the thermal plants in the system.

That is,

$$C_T = \sum_{i=1}^{M} C_i(P_{Ti}, t) \tag{2.14}$$

or

$$C_T = C_T (P_{T1}, P_{T2}, \ldots., P_{TM}, t) \tag{2.15}$$

2-4 Transmission Line Losses

Due to the development of large integrated power systems and the interconnections between networks, it is necessary to consider not only the cost of operating the generating units but also the cost of transmitting electrical power from the generating stations to the loads. This is taken care of by considering the power losses in the transmission lines while formulating the problem.

From the point of view of mathematical formulation, it is desirable to represent the power losses in the system as a function of the source outputs. It is done by using the approximate transmission loss formula [18]. The formula is,

$$P_L = \sum_{x=1}^{N+M} \sum_{y=1}^{N+M} P_x B_{xy} P_y \tag{2.16}$$

where

B_{xy} = Loss formula coefficients.

P_x, P_y = Source powers (output of the M thermal and N hydroplants)

2-5 System Restrictions

The total cost of operating the system is to be minimized subject to the following constraints:

(i) The total generation of the system should be equal to the system load demand plus the transmission losses in the system.

That is,

$$\sum_{i=1}^{M} P_{Ti} \quad + \quad \sum_{j=1}^{N} P_{Hj} \quad = \quad P_D + P_L \qquad (2.17)$$

This equation must be satisfied at all instants during the optimization interval.

(ii) The amount of water to be used by each hydroplant, during the optimization interval, is predetermined and limited. That is,

$$\int_o^T q_j \, dt = Q_j \quad = \quad \text{constant} \qquad (2.18)$$

$$j \quad = \quad 1, 2, \ldots, N$$

(iii) Besides the above-mentioned constraints, the economic schedule must conform to other design

and functional requirements. These can be classified as,

(a) Design limitations, and

(b) Functional limitations.

The design limitations appear because of the reservoir and plant characteristics. The size of the dam, the reservoir, the penstocks, the turbines, etc., determine these limitations.

The limits on maximum discharge due to the design considerations can be written as,

$$q_j \leq q_{j_{\max D}} \qquad (2.19)$$

The value of $q_{j_{\max D}}$ depends upon the design of the penstocks and the turbines.

The forebay elevation is restricted to a definite range. The minimum and maximum values are determined by the design of the dam, the reservoir and the location of the intakes. That is,

$$h_{Fj_{\min D}} \leq h_{Fj} \leq h_{Fj_{\max D}} \qquad (2.20)$$

$$j = 1, 2, \ldots, N.$$

The forebay elevation is directly related to the storage, hence equation (2.20) can also be interpreted as,

$$s_{j_{min\ D}} \leq s_j \leq s_{j_{max\ D}} \tag{2.21}$$

$$j = 1, 2, ..., N$$

The functional limitations arise because of the fact that most of the hydro-electric plants are multi-purpose in nature; hence it may be necessary to maintain certain discharges and storage levels to meet obligations other than power generation. These requirements may be seasonal or regular, but for any generation schedule to be admissible, these conditions must be satisfied. Some of these restrictions are given below.

Maximum forebay elevation due to flood prospect,

$$h_{Fj} \leq h_{Fj_{max}} \text{ (flood)} \tag{2.22}$$

$$j = 1, 2, ..., N$$

Minimum plant discharge and spillage to meet the irrigational obligations,

$$q_j + \sigma_j \geq q_{j_{min}} \text{ (Irrigation)} \tag{2.23}$$

$$j = 1, 2, ..., N$$

Minimum plant discharge and spillage required to meet the navigational commitments,

$$q_j + \sigma_j \geq q_{j_{min}} \text{ (Navigation)} \tag{2.24}$$

The storages and the discharges may be further restricted by project limitations other than those described above.

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CHAPTER III

SCHEDULING EQUATIONS FOR A HYDRO-THERMAL SYSTEM - HYDRO-PLANTS ON SEPARATE STREAMS

3-1 Statement of The Problem

Consider a hydro-thermal power system containing M thermal and N hydro-electric power plants, interconnected electrically by a network of transmission lines. The hydro-electric plants are located on separate streams. The load demand and the natural inflows are known functions of time. It is required to obtain equations for computing the generation schedule for each of the power plants. This schedule must be such that the operating cost of the system over the optimization interval is minimized, while the load demand is met. The transmission losses and the effect of variation of head upon the hydro-plant characteristics are to be taken into consideration. Also, the system restrictions explained in section 2-5 should not be violated.

3-2 Review of the Current Methods

A considerable amount of work has been done to obtain general methods and equations which provide the desired economic schedules. A large number of these works are listed in references [1,2] of this thesis. Some of the important scheduling equations are those derived by Ricard [12], Chandler, Dandeno, Glimn and Kirchmayer [13], Cypser [5], Carey [19], Watchcorn [4] and

Glimn and Kirchmayer [14]. It has been shown by Glimn and Kirchmayer [14] and Arismunandar [1] that these equations are essentially equivalent to each other. In terms of the symbols adopted in this thesis, Glimn and Kirchmayer [14] equations are,

(i) For the i^{th} thermal plant,

$$\frac{\partial C_T}{\partial P_{Ti}} + \lambda \frac{\partial P_L}{\partial P_{Ti}} = \lambda \tag{3.1}$$

$$i = 1, 2, \ldots, M$$

(ii) For the j^{th} hydro-electric plant,

$$\gamma_{j0} \; e^{\int_0^t \frac{\partial q_j}{\partial h_j} \frac{dt}{Aj}} \; \frac{\partial q_j}{\partial P_{Hj}} + \lambda \frac{\partial P_L}{\partial P_{H_j}} = \lambda \tag{3.2}$$

$$j = 1, 2, \ldots, N$$

<u>3-3</u> <u>Mathematical Formulation</u>

The problem stated in section 3-1 will be formulated in the following paragraphs.

The cost function is constructed as explained in section 2-3. The time integral of this function is to be minimized over the optimization interval, i.e.,

$$I = \int_{0}^{T} C_T \; (P_{T1}, P_{T2}, \ldots, P_{TM}, t) \; dt \qquad (3.3)$$

is to be minimized, subject to the following constraints:

(i) The total generation of the system must be equal to the load demand plus the transmission losses in the system, i.e.,

$$\sum_{i=1}^{M} P_{Ti} + \sum_{j=1}^{N} P_{Hj} = P_D + P_L \qquad (3.4)$$

This equation can be rewritten as,

$$\phi = P_D + P_L - \sum_{i=1}^{M} P_{Ti} - \sum_{j=1}^{N} P_{Hj} \equiv 0 \qquad (3.5)$$

This equation must be satisfied at all instants during the optimization interval.

(ii) The total amount of water to be used by each hydro-plant, over the optimization interval, is predetermined and limited, and is given by,

$$\int_{0}^{T} q_j \; dt = Q_j = \text{constant}$$

$$j = 1, 2, \ldots, N \qquad (3.6)$$

(iii) Various design and functional limitations (explained in detail in section 2-5) when combined together, result in putting upper and lower limits on the storages, discharges and the power outputs of the hydro-electric power plants. These can be expressed as follows:

Discharge limits:

$$\underline{q}_j \leq q_j \leq \overline{q}_j \tag{3.7}$$

$\underline{q}_j$ and $\overline{q}_j$ represent the lower and upper limits on q_j.

Storage limits:

$$\underline{s}_j \leq s_j \leq \overline{s}_j \tag{3.8}$$

$$j = 1, 2, \ldots, N$$

The limits on storages and discharges result in limiting the power output of the hydro-electric plants. These can be expressed as,

$$\underline{P}_{Hj} \leq P_{Hj} \leq \overline{P}_{Hj} \tag{3.9}$$

$$j = 1, 2, \ldots, N$$

Hence I in (3.3) is to be minimized subject to the constraints given by equations (3.5), (3.6), (3.7), (3.8) and (3.9). The two-sided inequality constraints given by (3.7), (3.8) and (3.9) can be handled by transforming

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them into equality constraints [9]. This is done by introducing a new variable for each of the inequality constraints. Such manipulations will increase the number of variables of the system considerably (which ultimately will increase the number of Euler equations for the system by the same number). For this reason these constraints are not considered while formulating the problem.

For the variational formulation, only the constraint given by equation (3.5) will be considered. The minimization of I subject to this constraint is a Lagrange problem in the Calculus of Variations. The problem is solved by replacing the cost function by an augmented function H [7,9], defined by,

$$H = C_T + \lambda \phi \tag{3.10}$$

That is,

$$H = C_T + \lambda \left[P_D + P_L - \sum_{i=1}^{M} P_{Ti} - \sum_{j=1}^{N} P_{Hj} \right] \tag{3.11}$$

where λ is a Lagrange multiplier.

C_T is given by equation (2.15), while P_L and P_{Hj} are defined by equations (2.16) and (2. 7) respectively. Thus, it can be seen that H is a function of the type,

$$H = H(P_{T1}, P_{T2}, \ldots, P_{TM}, s_1, s_2, \ldots, s_N, \dot{s}_1, \dot{s}_2, \ldots, \dot{s}_N, \lambda, t) \tag{3.12}$$

Hence, for maximum economy, minimize

$$J = \int_o^T H(P_{T1}, P_{T2}, \ldots, P_{TM}, s_1, s_2, \ldots, s_N, \dot{s}_1, \dot{s}_2, \ldots, \dot{s}_N, \lambda, t)\, dt \quad (3.13)$$

The extremal arcs are obtained by solving the Euler equations for the system. Also, for any extremal to be admissible, the constraints given by equations (3.5), (3.6), (3.7), (3.8) and (3.9) must be satisfied. In order to investigate whether the functional J attained a minimum, it would be desirable to check that the other necessary and sufficient conditions are satisfied [1, 8]. However, it is difficult to do so in practice.

Thus the scheduling equations for the power plants are given by the Euler equations for the system.

The Euler equations for the system are:

(i) For the i^{th} thermal plant,

$$\frac{\partial H}{\partial P_{Ti}} = 0 \qquad i = 1, 2, \ldots, M \quad (3.14)$$

(ii) For the j^{th} hydro-electric plant,

$$\frac{\partial H}{\partial s_j} - \frac{d}{dt} \frac{\partial H}{\partial \dot{s}_j} = 0 \qquad j = 1, 2, \ldots, N \quad (3.15)$$

The Euler equation for the hydro-electric plants can be transformed into a more convenient form, as shown in the following section.

3-4 Euler Equation in the Modified Form

The coordinating equations for the hydro-electric plants are the Euler equations. These equations are put into a more convenient integro-differential form.

The scheduling equation for the j^{th} hydro-electric plant is,

$$\frac{\partial H}{\partial s_j} - \frac{d}{dt}\frac{\partial H}{\partial \dot{s}_j} = 0 \tag{3.16}$$

or

$$\frac{\partial H}{\partial s_j} = \frac{d}{dt}\frac{\partial H}{\partial \dot{s}_j} \tag{3.17}$$

Let us define

$$Z_j(t) = \frac{\partial H}{\partial s_j} \Big/ \frac{\partial H}{\partial \dot{s}_j} \tag{3.18}$$

only when $\frac{\partial H}{\partial \dot{s}_j}$ is not equal to zero.

Note that in this thesis it is assumed that $\frac{\partial P_{Hj}}{\partial \dot{s}_j}$ is non-zero at all times.

Hence

$$\frac{\partial H}{\partial \dot{s}_j} = \frac{\partial H}{\partial \dot{s}_j} \cdot Z_j(t) \tag{3.19}$$

Now, from equations (3.17) and (3.19), we get

$$\frac{\partial H}{\partial s_j} \cdot Z_j(t) = \frac{d}{dt}\left[\frac{\partial H}{\partial \dot{s}_j}\right] \tag{3.20}$$

or

$$Z_j(t)\ dt = \frac{d\left[\dfrac{\partial H}{\partial \dot{s}j}\right]}{\left[\dfrac{\partial H}{\partial \dot{s}_j}\right]} \tag{3.21}$$

Integrating both sides, we get

$$\int_0^t Z_j(t)\ dt = \log_e \left[\frac{\partial H}{\partial \dot{s}_j}\right] + K_j \tag{3.22}$$

where, K_j is an arbitrary constant of integration.

Let

$$K_j = \log_e \left[\frac{1}{\gamma_{j0}}\right] \tag{3.23}$$

then, equation (3.22) can be rewritten as,

$$\int_0^t Z_j(t)\ dt = \log_e \left[\frac{1}{\gamma_{j0}} \cdot \frac{\partial H}{\partial \dot{s}_j}\right] \tag{3.24}$$

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or

$$\frac{1}{\gamma_{j0}} \cdot \frac{\partial H}{\partial \dot{s}_j} = e^{\int_0^T Z_j(t)\, dt} \qquad (3.25)$$

or

$$\frac{\partial H}{\partial \dot{s}_j} = \gamma_{j0} \cdot e^{\int_0^t Z_j(t)\, dt} \qquad (3.26)$$

where, $Z_j(t)$ is defined by equation (3.18) and γ_{j0} is an arbitrary constant.

This equation is equivalent to the Euler equation and will be called the "modified" Euler equation in the remainder of this thesis.

3-5 Derivation of the Scheduling Equations

The scheduling equations for the thermal and the hydro-electric plants of the system are given by equations (3.14) and (3.15) respectively.

(i) For the i^{th} thermal plant, the scheduling equation is,

$$\frac{\partial H}{\partial P_{Ti}} = 0$$

where H is defined by equation (3.11). Hence, we get

$$\frac{\partial C_T}{\partial P_{Ti}} + \lambda \frac{\partial P_L}{\partial P_{Ti}} - \lambda = 0 \qquad (3.27)$$

or

$$\frac{\partial C_T}{\partial P_{Ti}} + \lambda \frac{\partial P_L}{\partial P_{Ti}} = \lambda \qquad (3.28)$$

(ii) For the hydro-electric plants, the scheduling equations are obtained by using the "modified" Euler equation given by equation (3.26).

Hence, the scheduling equation for the j^{th} hydro-electric plant is:

$$\frac{\partial H}{\partial \dot{s}_j} = \gamma_{j0} \cdot e^{\int_0^t Z_j(t)\, dt} \qquad (3.29)$$

where, $Z_j(t)$ is given by equation (3.18).

H is defined by the equation (3.11), while P_L and P_{Hj} are given by equations (2.16) and (2.7) respectively.

Now taking the partial derivative of H with respect to $\dot{s}_j$ and substituting into equation (3.26), we get

$$\lambda \left[\frac{\partial P_L}{\partial \dot{s}_j} - \frac{\partial P_{Hj}}{\partial \dot{s}_j} \right] = \gamma_{j0} \; e^{\int_0^t Z_j(t)\, dt} \qquad (3.30)$$

But,

$$\frac{\partial P_L}{\partial \dot{s}_j} = \frac{\partial P_L}{\partial P_{Hj}} \cdot \frac{\partial P_{Hj}}{\partial \dot{s}_j} \qquad (3.31)$$

and putting,

$$\gamma_{j0}\, e^{\int_0^t Z_j(t)\,dt} = \gamma_j(t) \tag{3.32}$$

in equation (3.30), we get,

$$\lambda \left[\frac{\partial P_L}{\partial P_{Hj}} \cdot \frac{\partial P_{Hj}}{\partial \dot{s}_j} - \frac{\partial P_{Hj}}{\partial \dot{s}_j}\right] = \gamma_j(t) \tag{3.33}$$

or

$$\lambda \frac{\partial P_{Hj}}{\partial \dot{s}_j} \left[\frac{\partial P_L}{\partial P_{Hj}} - 1\right] = \gamma_j(t) \tag{3.34}$$

or

$$\lambda \left[\frac{\partial P_L}{\partial P_{Hj}} - 1\right] = \gamma_j(t) \cdot \frac{1}{\frac{\partial P_{Hj}}{\partial \dot{s}_j}} \tag{3.35}$$

Now, consider

$$\frac{\partial \dot{s}_j}{\partial P_{Hj}} = \frac{1}{\frac{\partial P_{Hj}}{\partial \dot{s}_j}} \tag{3.36}$$

This equation is true if the derivatives $\frac{\partial P_{Hj}}{\partial \dot{s}_j}$ and $\frac{\partial \dot{s}_j}{\partial P_{Hj}}$ exist and are non-zero. A proof is given by Glimn and Kirchmayer [14].

Hence, equation (3.35) can be rewritten as,

$$\lambda \left[\frac{\partial P_L}{\partial P_{Hj}} - 1\right] = \gamma_j(t) \cdot \frac{\partial \dot{s}_j}{\partial P_{Hj}} \tag{3.37}$$

Rearranging the terms, we get

$$\gamma_j(t) \cdot \left[-\frac{\partial \dot{s}_j}{\partial P_{Hj}}\right] + \lambda \frac{\partial P_L}{\partial P_{Hj}} = \lambda \tag{3.38}$$

$\gamma_j(t)$ is given by equation (3.32), while $Z_j(t)$ is given by equation (3.18) as,

$$Z_j(t) = \frac{\dfrac{\partial H}{\partial s_j}}{\dfrac{\partial H}{\partial \dot{s}_j}}$$

In order to evaluate $Z_j(t)$, the values of $\frac{\partial H}{\partial s_j}$ and $\frac{\partial H}{\partial \dot{s}_j}$ are to be obtained.

Taking the partial derivative of H with respect to s_j, we get

$$\frac{\partial H}{\partial s_j} = \lambda \left[\frac{\partial P_L}{\partial s_j} - \frac{\partial P_{Hj}}{\partial s_j}\right] \tag{3.39}$$

but,

$$\frac{\partial P_L}{\partial s_j} = \frac{\partial P_L}{\partial P_{Hj}} \cdot \frac{\partial P_{Hj}}{\partial s_j} \tag{3.40}$$

hence

$$\frac{\partial H}{\partial s_j} = \lambda \left[\frac{\partial P_L}{\partial P_{Hj}} \cdot \frac{\partial P_{Hj}}{\partial s_j} - \frac{\partial P_{Hj}}{\partial s_j} \right] \tag{3.41}$$

or

$$\frac{\partial H}{\partial s_j} = \lambda \frac{\partial P_{Hj}}{\partial s_j} \left[\frac{\partial P_L}{\partial P_{Hj}} - 1 \right] \tag{3.42}$$

Now, taking the partial derivative of H with respect to $\dot{s}_j$, we obtain

$$\frac{\partial H}{\partial \dot{s}_j} = \lambda \left[\frac{\partial P_L}{\partial \dot{s}_j} - \frac{\partial P_{Hj}}{\partial \dot{s}_j} \right] \tag{3.43}$$

Substituting for $\frac{\partial P_L}{\partial \dot{s}_j}$ from equation (3.31) in equation (3.43) and rearranging the terms, we get

$$\frac{\partial H}{\partial \dot{s}_j} = \lambda \frac{\partial P_{Hj}}{\partial \dot{s}_j} \left[\frac{\partial P_L}{\partial P_{Hj}} - 1 \right] \tag{3.44}$$

Now, from equations (3.42), (3.44) and (3.18) we obtain

$$z_j(t) = \frac{\lambda \dfrac{\partial P_{Hj}}{\partial s_j} \left[\dfrac{\partial P_L}{\partial P_{Hj}} - 1 \right]}{\lambda \dfrac{\partial P_{Hj}}{\partial \dot{s}_j} \left[\dfrac{\partial P_L}{\partial P_{Hj}} - 1 \right]} \tag{3.45}$$

or

$$Z_j(t) = \frac{\dfrac{\partial P_{Hj}}{\partial s_j}}{\dfrac{\partial P_{Hj}}{\partial \dot{s}_j}} \tag{3.46}$$

Hence from equations (3.38), (3.32) and (3.46), we get the scheduling equation for the j^{th} hydro-electric plant as,

$$\gamma_{j0} \cdot e^{\int_0^t \left[\frac{\partial P_{Hj}}{\partial s_j} / \frac{\partial P_{Hj}}{\partial \dot{s}_j}\right] dt} \cdot \left[-\frac{\partial \dot{s}_j}{\partial P_{Hj}}\right] + \lambda \frac{\partial P_L}{\partial P_{Hj}} = \lambda \tag{3.47}$$

3-6 Comparison with the Scheduling Equations Derived by Glimn and Kirchmayer [14]

Their scheduling equation for the i^{th} thermal plant is given by equation (3.1). This is identical to that derived in this thesis (given by equation (3.28)).

For the j^{th} hydro-electric plant, their scheduling equation is given by equation (3.2).

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Also, they have shown in their paper [14] that

$$\frac{\partial q_j}{\partial h_j} = \frac{\dfrac{\partial P_{Hj}}{\partial h_j}}{-\dfrac{\partial P_{Hj}}{\partial q_j}} \tag{3.48}$$

Thus, equation (3.2) can be rewritten as,

$$\gamma_{j0} \; e^{\int_0^t \left[\frac{\partial P_{Hj}}{\partial h_j} / - \frac{\partial P_{Hj}}{\partial q_j}\right] \frac{dt}{Aj}} \cdot \frac{\partial q_j}{\partial P_{Hj}} + \lambda \frac{\partial P_L}{\partial P_{Hj}} = \lambda \tag{3.49}$$

Now, comparing the equations (3.47) and (3.49), we find that the equations are identical only if the following relations are true:

$$- \frac{\partial \dot{s}_j}{\partial P_{Hj}} = \frac{\partial q_j}{\partial P_{Hj}} \tag{3.50}$$

and,

$$\frac{1}{A_j} \cdot \frac{\partial P_{Hj}}{\partial h_j} = \frac{\partial P_{Hj}}{\partial s_j} \tag{3.51}$$

The discharge of a plant can be represented as the difference between the natural inflow and the rate of change of storage in the plant's reservoir, i.e.,

$$q_j = F_j - \dot{s}_j \tag{3.52}$$

also, we know that,

$$P_{Hj} = P_{Hj}(q_j, h_j, t) \tag{3.53}$$

hence, it can be concluded that both q_j and $(F - \dot{s}_j)$ are functions of P_{Hj} and h_j. F_j is an uncontrollable variable and is also independent of P_{Hj}. By taking the partial derivatives of both sides of equation (3.52), we get

$$\frac{\partial q_j}{\partial P_{Hj}} = - \frac{\partial \dot{s}_j}{\partial P_{Hj}} \tag{3.54}$$

Hence, equation (3.50) is true.

Furthermore, if the tail race elevation and the losses in the conduits are taken to be constant, and if the surface area of the reservoir of the j^{th} hydroelectric plant, A_j, is a constant, then the net head at the plant can be written as,

$$h_j = \frac{s_j}{A_j} + K_{1j} \tag{3.55}$$

where K_{1j} is a constant.

Again,

$$\begin{aligned} P_{Hj} &= P_{Hj}\ (q_j,\ h_j,\ t) \\ &= P_{Hj}\left[q_j,\ \left(\frac{s_j}{A_j} + K_{1j}\right),\ t\right] \end{aligned} \tag{3.56}$$

Therefore

$$\frac{\partial P_{Hj}}{\partial h_j} = A_j \frac{\partial P_{Hj}}{\partial sj} \tag{3.57}$$

or

$$\frac{1}{A_j} \frac{\partial P_{Hj}}{\partial h_j} = \frac{\partial P_{Hj}}{\partial s_j} \tag{3.58}$$

Therefore, equation (3.51) is true.

Thus, the scheduling equation (3.47), derived in this thesis for the j^{th} hydro-plant, is equivalent to that of Glimn and Kirchmayer [14].

CHAPTER IV

SCHEDULING EQUATIONS FOR A HYDRO-THERMAL SYSTEM - SOME OF THE HYDRO-PLANTS LOCATED ON COMMON STREAMS

4-1 Statement of the Problem

The power system under study consists of M thermal and N hydro-electric power plants, interconnected electrically by a net-work of transmission lines. For simplicity, it is assumed that all except two hydro-plants are located on separate streams. Let the k^{th} and $k+1^{st}$ be the two hydro-plants located on the same stream. We assume that the load demand and the natural inflows are known functions of time. The time taken by the water to flow from the upstream plant to the downstream plant is τ. It is desired to obtain the scheduling equations, such that the most economical generation schedule can be obtained.

4-2 Review of Current Methods

In 1941, Burr [20] made an attempt to develop a general method for determining the most economical loading of the common-flow hydro-electric plants. Using the Steepest Descent method, Cypser [5] solved the Euler equations for a system in which the two hydro-electric plants are located on the same stream. Menon [15] solved the common-flow problem by constructing a

series of minimizing sequences. Kirchmayer [18] extended the incremental cost technique to the hydro-plants located on common water-shed. The hydro-plants were treated as constant head plants. Drake, Kirchmayer, Mayall and Wood [21] used that method to coordinate the expenditure of the steam and hydro-electric resources of the South California Edision Company. The effect of variation of head upon the hydro-plant characteristics was not taken into consideration.

4-3 Mathematical Formulation

The problem stated in section 4-1 is similar in nature to that stated in section 3-1. The only difference is that the k^{th} and $k+1^{st}$ hydro-plants are located on the same stream. The following analysis is quite general in nature and can be extended to a system in which more than two hydro-plants may be located on the same stream.

The cost function for the system is constructed as explained in section 2-3. The time integral of this function is to be minimized over the optimization interval. That is, we minimize

$$I = \int_0^T C_T \, (P_{T1}, P_{T2}, \ldots, P_{TM}, t) \, dt \qquad (4.1)$$

subject to the constraints listed in section 2-5.

The only constraint which will be considered here is that the total generation of the system must be equal to the load demand plus the transmission losses in the system, that is,

$$\sum_{i=1}^{M} P_{Ti} + \sum_{j=1}^{N} P_{Hj} = P_D + P_L \qquad (4.2)$$

or

$$\phi = P_D + P_L - \sum_{i=1}^{M} P_{Ti} - \sum_{0}^{N} P_{Hj} \equiv 0 \qquad (4.3)$$

However, for any optimal solution to be admissible all other constraints described in section 2-5 must be taken into account.

Again, minimization of I in (4.1) subject to the constraint given by equation (4.3), is a Lagrange problem in the Calculus of Variations. It is handled by replacing C_T by an augmented function H [7,9], defined by,

$$H = C_T + \lambda \phi$$

$$= C_T + \lambda (P_D + P_L - \sum_{i=1}^{M} P_{Ti} - \sum_{j=1}^{N} P_{Hj}) \qquad (4.4)$$

From equations (2.16), (2.7) and (2.11), it is seen that H is a function of the type,

$$H = H(P_{T1}, P_{T2}, \ldots, P_{TM}, s_1, s_2, \ldots, s_N, \dot{s}_1, \dot{s}_2, \ldots, \dot{s}_N, \lambda, \tau) \qquad (4.5)$$

Now, for maximum economy, we minimize

$$J = \int_0^T H \, . \, dt \tag{4.6}$$

The optimal solution is obtained by solving the Euler equations for the system. The Euler equations for the system are,

$$\frac{\partial H}{\partial P_{Ti}} = 0 \tag{4.7}$$

$$i = 1, 2, \ldots, N$$

and

$$\frac{\partial H}{\partial s_j} - \frac{d}{dt} \frac{\partial H}{\partial \dot{s}_j} = 0 \tag{4.8}$$

$$j = 1, 2, \ldots, N$$

Equation (4.8) can be transformed into a more convenient form as given by equation (3.26) and will be used for obtaining the required scheduling equations.

<u>4-4</u> <u>Derivation of the Scheduling Equations</u>

The scheduling equations for thermal and the hydro-electric plants are obtained by using equations (4.7) and (3.26) respectively.

(a) Taking the partial derivative of H (defined by equation (4.4)) with respect to P_{Ti} and substituting into equation (4.7), we get,

$$\frac{\partial C_T}{\partial P_{Ti}} + \lambda \frac{\partial P_L}{\partial P_{Ti}} - \lambda = 0 \qquad (4.9)$$

or

$$\frac{\partial C_T}{\partial P_{Ti}} + \lambda \frac{\partial P_L}{\partial P_{Ti}} = \lambda \qquad (4.10)$$

This equation is identical to that for the i^{th} thermal plant of the system discussed in Chapter-3.

(b) The scheduling equations for the hydro-plants are obtained by using the "modified" Euler equation (3.26).

(i) The hydro-electric plants situated on separate streams:

The coordinating equations can be derived as in section 3-5. The coordinating equation for the j^{th} hydro-plant is (refer to equation (3.47)).

$$\gamma_{j0}\, e^{\int_0^t \left[\frac{\partial P_{Hj}}{\partial s_j} \Big/ \frac{\partial P_{Hj}}{\partial \dot{s}_j}\right] dt} \cdot \left[-\frac{\partial \dot{s}_j}{\partial P_{Hj}}\right] + \lambda \frac{\partial P_L}{\partial P_{Hj}} = \lambda \qquad (4.11)$$

$j = 1, 2, \ldots, k-1, k+2, \ldots, N$

$j \neq k, k+1$

(ii) The downstream, $k+1^{st}$, hydro-electric plant:

The coordinating equation is obtained by taking the partial derivatives of H with respect to s_{k+1} and $\dot{s}_{k+1}$ and substituting them in the modified Euler equation (3.26). Hence, the scheduling equation for the hydro-plant is,

$$\lambda\left[\frac{\partial P_L}{\partial \dot{s}_{k+1}} - \frac{\partial P_{Hk+1}}{\partial \dot{s}_{k+1}}\right] = \gamma_{(k+1)0} \cdot e^{\int_0^t Z_{k+1}(t)\, dt} \qquad (4.12)$$

where $Z_k(t)$ is given by equation (3.18).

Also,

$$\frac{\partial P_L}{\partial \dot{s}_{k+1}} = \frac{\partial P_L}{\partial P_{Hk+1}} \cdot \frac{\partial P_{Hk+1}}{\partial \dot{s}_{k+1}} \qquad (4.13)$$

because $\dot{s}_{k+1}$ appears in the expression for P_{Hk+1} only (refer to equation (2.11)).

Substituting for $\dfrac{\partial P_L}{\partial \dot{s}_{k+1}}$ in equation (4.12) we get

$$\lambda\left[\frac{\partial P_L}{\partial P_{Hk+1}} \cdot \frac{\partial P_{Hk+1}}{\partial \dot{s}_{k+1}} - \frac{\partial P_{Hk+1}}{\partial \dot{s}_{k+1}}\right] = \gamma_{(k+1)0} \cdot e^{\int_0^t Z_{k+1}(t)\, dt} \qquad (4.14)$$

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or

$$\lambda \frac{\partial P_{Hk+1}}{\partial \dot{s}_k} \left[\frac{\partial P_L}{\partial P_{Hk+1}} - 1 \right] = \gamma_{(k+1)0} \, e^{\int_0^t Z_{k+1}(t)\, dt} \tag{4.15}$$

also, it can be shown [14] that

$$\frac{\partial P_{Hk+1}}{\partial \dot{s}_{k+1}} = \frac{1}{\dfrac{\partial \dot{s}_{k+1}}{\partial P_{Hk+1}}} \tag{4.16}$$

Hence, equation (4.15) can be rewritten as,

$$\gamma_{(k+1)0} \cdot e^{\int_0^t Z_{k+1}(t)\, dt} \cdot \left[- \frac{\partial \dot{s}_{k+1}}{\partial P_{Hk+1}} \right] + \lambda \frac{\partial P_L}{\partial P_{Hk+1}} = \lambda \tag{4.17}$$

In order to find the expression for $Z_{k+1}(t)$, the partial derivatives of H with respect to s_{k+1} and $\dot{s}_{k+1}$ are to be evaluated.

$$\frac{\partial H}{\partial s_{k+1}} = \lambda \left[\frac{\partial P_L}{\partial s_{k+1}} - \frac{\partial P_{Hk+1}}{\partial s_{k+1}} \right] \tag{4.18}$$

but,

$$\frac{\partial P_L}{\partial s_{k+1}} = \frac{\partial P_L}{\partial P_{Hk+1}} \cdot \frac{\partial P_{Hk+1}}{\partial s_{k+1}} \tag{4.19}$$

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Hence, substituting (4.19) into (4.17) we get

$$\frac{\partial H}{\partial s_{k+1}} = \lambda \frac{\partial P_{Hk+1}}{\partial s_{k+1}} \left[\frac{\partial P_L}{\partial s_{k+1}} - 1 \right] \qquad (4.20)$$

Similarly, we obtain

$$\frac{\partial H}{\partial \dot{s}_{k+1}} = \lambda \frac{\partial P_{Hk+1}}{\partial \dot{s}_{k+1}} \left[\frac{\partial P_L}{\partial \dot{s}_{k+1}} - 1 \right] \qquad (4.21)$$

Substituting for $\frac{\partial H}{\partial s_{k+1}}$ and $\frac{\partial H}{\partial \dot{s}_{k+1}}$ in equation (3.18),

we get

$$Z_{k+1}(t) = \frac{\lambda \dfrac{\partial P_{Hk+1}}{\partial s_{k+1}} \left[\dfrac{\partial P_L}{\partial P_{Hk+1}} - 1 \right]}{\lambda \dfrac{\partial P_{Hk+1}}{\partial \dot{s}_{k+1}} \left[\dfrac{\partial P_L}{\partial P_{Hk+1}} - 1 \right]} \qquad (4.22)$$

or

$$Z_{k+1}(t) = \frac{\dfrac{\partial P_{Hk+1}}{\partial s_{k+1}}}{\dfrac{\partial P_{Hk+1}}{\partial \dot{s}_{k+1}}} \qquad (4.23)$$

Hence, from equations (4.17) and (4.23), the scheduling equation for the k+1st hydro-plant is

$$\gamma_{(k+1)0} \, . e^{\int_0^t \left[\frac{\partial P_{Hk+1}}{\partial s_{k+1}} / \frac{\partial P_{Hk+1}}{\partial \dot{s}_{k+1}} \right] dt} . \left[- \frac{\partial \dot{s}_{k+1}}{\partial P_{Hk+1}} \right] + \lambda \frac{\partial P_L}{\partial P_{Hk+1}} = \lambda$$

(4.24)

This equation is identical to the scheduling equation for the hydro-plants located on separate streams (equation (4.11)).

(iii) The upstream, k^{th}, hydro-electric plant:

The water released from the upstream plant at an instant $(t-\tau)$ reaches the downstream plant at an instant t. This discharge flows into the reservoir of the downstream plant and thus affects the operation of that plant. This has been discussed in detail in section 2-2. There it has been shown that the variable $\dot{s}_k$ appears in the expressions for both P_{Hk} and P_{Hk+1} (refer to equations (2.8) and (2.11)), while the variable s_k appears only in P_{Hk}. This must be kept in mind when taking the partial derivatives of H with respect to s_k and $\dot{s}_k$.

The scheduling equation for the k^{th} plant is obtained by taking the partial derivatives of H with respect to s_k and $\dot{s}_k$, and substituting them into the "modified" Euler equation (equation (3.26)).

Thus, we get

$$\lambda \left[\frac{\partial P_L}{\partial \dot{s}_k} - \frac{\partial P_{Hk}}{\partial \dot{s}_k} - \frac{\partial P_{Hk+1}}{\partial \dot{s}_k} \right] = \gamma_{k0}\, e^{\int_0^t Z_k(t)\, dt} \qquad (4.25)$$

where, $Z_k(t)$ is given by equation (3.18).

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Also, we can write

$$\frac{\partial P_L}{\partial \dot{s}_k} = \frac{\partial P_L}{\partial P_{Hk}} \cdot \frac{\partial P_{Hk}}{\partial \dot{s}_k} + \frac{\partial P_L}{\partial P_{Hk+1}} \cdot \frac{\partial P_{Hk+1}}{\partial \dot{s}_k} \tag{4.26}$$

because P_L is a function of both P_{Hk} and P_{Hk+1}; while both P_{Hk} and P_{Hk+1} are functions of $\dot{s}_k$.

Hence, equation (4.25) can be rewritten as,

$$\lambda \left[\frac{\partial P_L}{\partial P_{Hk}} \cdot \frac{\partial P_{Hk}}{\partial \dot{s}_k} + \frac{\partial P_L}{\partial P_{Hk+1}} \cdot \frac{\partial P_{Hk+1}}{\partial \dot{s}_k} - \frac{\partial P_{Hk}}{\partial \dot{s}_k} - \frac{\partial P_{Hk+1}}{\partial \dot{s}_k} \right] = \gamma_{k0} \cdot e^{\int_0^t Z_k(t)\, dt} \tag{4.27}$$

Now, putting

$$\gamma_{k0} \cdot e^{\int_0^t Z_k(t)\, dt} = \gamma_k(t) \tag{4.28}$$

and rearranging the terms, we get

$$\lambda \frac{\partial P_{Hk}}{\partial \dot{s}_k} \left[\frac{\partial P_L}{\partial P_{Hk}} - 1 \right] + \lambda \frac{\partial P_{Hk+1}}{\partial \dot{s}_k} \left[\frac{\partial P_L}{\partial P_{Hk+1}} - 1 \right] = \gamma_k(t) \tag{4.29}$$

or

$$\lambda \frac{\partial P_{Hk}}{\partial \dot{s}_k} \left[\frac{\partial P_L}{\partial P_{Hk}} - 1 \right] = \gamma_k(t) - \lambda \frac{\partial P_{Hk+1}}{\partial \dot{s}_k} \left[\frac{\partial P_L}{\partial P_{Hk+1}} - 1 \right] \tag{4.30}$$

We can write [14],

$$\frac{\partial P_{Hk}}{\partial \dot{s}_k} = \frac{1}{\dfrac{\partial \dot{s}_k}{\partial P_{Hk}}} \tag{4.31}$$

Hence, equation (4.30) can be written as,

$$\lambda \left[\frac{\partial P_L}{\partial PH_k} - 1\right] = \left[\gamma_k(t) - \lambda \frac{\partial P_{Hk+1}}{\partial \dot{s}_k} \left[\frac{\partial P_L}{\partial P_{Hk+1}} - 1\right]\right] \cdot \frac{\partial \dot{s}_k}{\partial P_{Hk}} \tag{4.32}$$

or

$$\left[\gamma_k(t) - \lambda \frac{\partial P_{Hk+1}}{\partial \dot{s}_k} \left[\frac{\partial P_L}{\partial P_{Hk+1}} - 1\right]\right] \cdot \left[- \frac{\partial \dot{s}_k}{\partial P_{Hk}}\right] + \lambda \frac{\partial P_L}{\partial P_{HL}} = \lambda \tag{4.33}$$

Rearranging the terms of the scheduling equation for the k+1st hydro-plant (equation (4.24)), we get

$$\lambda \left[\frac{\partial P_L}{\partial PH_{k+1}} - 1\right] = \gamma_{k+1}(t) \cdot \frac{\partial \dot{s}_{k+1}}{\partial P_{Hk+1}} \tag{4.34}$$

where

$$\gamma_{k+1}(t) = \gamma_{(k+1)0} \cdot e^{\int_0^t \left[\frac{\partial P_{Hk+1}}{\partial s_{k+1}} / \frac{\partial P_{Hk+1}}{\partial \dot{s}_{k+1}}\right] dt} \tag{4.35}$$

Now substituting for $\lambda \left[\frac{\partial P_L}{\partial P_{Hk+1}} - 1\right]$ in equation (4.33), we get,

$$\left[\gamma_k(t) - \gamma_{k+1}(t) \cdot \frac{\partial \dot{s}_{k+1}}{\partial P_{Hk+1}} \cdot \frac{\partial P_{Hk+1}}{\partial \dot{s}_k}\right] \cdot \left[- \frac{\partial \dot{s}_k}{\partial P_{Hk}}\right] + \lambda \frac{\partial P_L}{\partial P_{Hk}} = \lambda \qquad (4.36)$$

$\gamma_k(t)$ and $\gamma_{k+1}(t)$ are given by equation (4.28) and (4.35) respectively; and $Z_k(t)$ is given by equation (3.18). In order to evaluate $Z_k(t)$ we must find the partial derivatives of H with respect to s_k and $\dot{s}_k$.

The variable s_k appears only in P_{Hk}, hence, by taking the partial derivative of H with respect to s_k, we get,

$$\frac{\partial H}{\partial s_k} = \lambda \left[\frac{\partial P_L}{\partial s_k} - \frac{\partial P_{Hk}}{\partial s_k}\right] \qquad (4.37)$$

Also

$$\frac{\partial P_L}{\partial s_k} = \frac{\partial P_L}{\partial P_{Hk}} \cdot \frac{\partial P_{Hk}}{\partial s_k} \qquad (4.38)$$

therefore

$$\frac{\partial H}{\partial s_k} = \lambda \left[\frac{\partial P_L}{\partial P_{Hk}} \cdot \frac{\partial P_{Hk}}{\partial s_k} \quad \frac{\partial P_{Hk}}{\partial s_k}\right] \qquad (4.39)$$

or,

$$\frac{\partial H}{\partial s_k} = \lambda \frac{\partial P_{Hk}}{\partial s_k} \left[\frac{\partial P_L}{\partial P_{Hk}} - 1 \right] \tag{4.40}$$

Since both P_{Hk} and P_{Hk+1} are functions of $\dot{s}_k$, we obtain,

$$\frac{\partial H}{\partial \dot{s}_k} = \lambda \left[\frac{\partial P_L}{\partial \dot{s}_k} - \frac{\partial P_{Hk}}{\partial \dot{s}_k} - \frac{\partial P_{Hk+1}}{\partial \dot{s}_k} \right] \tag{4.41}$$

and using equation (4.26), we can write,

$$\frac{\partial H}{\partial \dot{s}_k} = \lambda \left[\frac{\partial P_L}{\partial P_{Hk}} \cdot \frac{\partial P_{Hk}}{\partial \dot{s}_k} + \frac{\partial P_L}{\partial P_{Hk+1}} \cdot \frac{\partial P_{Hk+1}}{\partial \dot{s}_k} - \frac{\partial P_{Hk}}{\partial \dot{s}_k} - \frac{\partial P_{Hk+1}}{\partial \dot{s}_k} \right] \tag{4.42}$$

rearranging the terms, we get

$$\frac{\partial H}{\partial \dot{s}_k} = \lambda \frac{\partial P_{Hk}}{\partial \dot{s}_k} \left[\frac{\partial P_L}{\partial P_{Hk}} - 1 \right] + \lambda \frac{\partial P_{Hk+1}}{\partial \dot{s}_k} \left[\frac{\partial P_L}{\partial P_{Hk+1}} - 1 \right] \tag{4.43}$$

Now, substituting for $\frac{\partial H}{\partial s_k}$ and $\frac{\partial H}{\partial \dot{s}_k}$ in equation (3.18), we get

$$Z_k(t) = \frac{\lambda \frac{\partial P_{Hk}}{\partial s_k} \left[\frac{\partial P_L}{\partial P_{Hk}} - 1 \right]}{\lambda \frac{\partial P_{Hk}}{\partial \dot{s}_k} \left[\frac{\partial P_L}{\partial P_{Hk}} - 1 \right] + \lambda \frac{\partial P_{Hk+1}}{\partial \dot{s}_k} \left[\frac{\partial P_L}{\partial P_{Hk+1}} - 1 \right]} \tag{4.44}$$

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or

$$Z_k(t) = \frac{\dfrac{\partial P_{Hk}}{\partial s_k} \left[\dfrac{\partial P_L}{\partial P_{Hk}} - 1\right]}{\dfrac{\partial P_{Hk}}{\partial \dot{s}_k}\left[\dfrac{\partial P_L}{\partial P_{Hk}} - 1\right] + \dfrac{\partial P_{Hk+1}}{\partial \dot{s}_k}\left[\dfrac{\partial P_L}{\partial P_{Hk+1}} - 1\right]} \tag{4.45}$$

Thus, the scheduling equation for the k^{th} plant is given by equation (4.36), where $\gamma_k(t)$ and $\gamma_{k+1}(t)$ are given by equations (4.28) and (4.35) respectively. Furthermore, $Z_k(t)$ is defined by equation (4.45).

4-5 Discussion of the Scheduling Equations for the Common-flow Plants

If we substitute

$$\gamma_k(t) - \gamma_{k+1}(t) \quad \frac{\partial \dot{s}_{k+1}}{\partial P_{Hk+1}} \cdot \frac{\partial P_{Hk+1}}{\partial \dot{s}_k} = \gamma_k' \tag{4.46}$$

in equation (4.36), we get

$$\gamma_k' \left[-\frac{\partial \dot{s}_k}{\partial P_{Hk}}\right] + \lambda \frac{\partial P_L}{\partial P_{Hk}} = \lambda \tag{4.47}$$

Comparing this equation with equation (4.24) for the $k+1^{st}$ hydro-plant, it is noticed that both the equations are similar in nature. The difference lies in the way γ_k' and γ_{k+1} vary with time.

γ_j for the j^{th} ($j \neq k$) hydro-plant is interpreted as the water conversion coefficient for the plant [14]. Its value determines the amount of water which is used over the optimization period. The influence of these coefficients on the scheduling equation is discussed in section 5-3. The water conversion coefficient for the up-stream plant is modified due to the presence of the $k+1^{st}$ (downstream) plant on the same stream. Thus if the value for $\gamma_{(k+1)0}$ is changed, the value for γ_k is also affected.

The time lag between the two reservoirs, τ, is involved in the expression for P_{Hk+1} due to $\dot{s}_k(t-\tau)$. In the scheduling equation, τ affects only the value of $\frac{\partial P_{Hk+1}}{\partial \dot{s}_k}$, and this appears only in the coordinating equations for the k^{th} hydro-electric plant.

Now, from equation (2.9), we have

$$P_{Hk+1}(t) = P_{Hk+1}\left(s_{k+1}(t), (\dot{s}_k(t-\tau) + \dot{s}_{k+1}(t))\right)$$

and as the variables of the system can be varied independently,

$$\frac{\partial P_{Hk+1}(t)}{\partial \dot{s}_{k+1}(t)} = \frac{\partial P_{Hk+1}(t)}{\partial \dot{s}_k(t-\tau)} \tag{4.48}$$

Also, $\dot{s}_k(t-\tau)$ can be written as some non-linear function of $\dot{s}_k(t)$ and τ, i.e.,

$$\dot{s}_k(t-\tau) = f\,(\dot{s}_k(t),\tau) \tag{4.49}$$

Then

$$\frac{\partial P_{Hk+1}(t)}{\partial \dot{s}_k(t)} = \frac{\partial P_{Hk+1}}{\partial \dot{s}_k(t-\tau)} \cdot \frac{\partial \dot{s}_k(t-\tau)}{\partial \dot{s}_k(t)} \tag{4.50}$$

Using equation (4.48), equation (4.50) can be written as,

$$\frac{\partial P_{Hk+1}}{\partial \dot{s}_k} = \frac{\partial P_{Hk+1}(t)}{\partial \dot{s}_{k+1}(t)} \cdot \frac{d\dot{s}_k(t-\tau)}{d\dot{s}_k(t)} \tag{4.51}$$

Now, if τ is constant, $\dot{s}_k(t-\tau)$ can be expanded using a Taylor's series, as

$$\dot{s}_k(t-\tau) = \dot{s}_k(t) - \tau\,\ddot{s}_k(t) + \frac{\tau^2}{2!}\,\dddot{s}\,(t) \cdots\cdots \tag{4.52}$$

Differentiating both the sides with respect to $\dot{s}_k(t)$, we get

$$\frac{d\dot{s}_k(t-\tau)}{d\dot{s}_k(t)} = 1 - \tau\,\frac{d\ddot{s}_k(t)}{d\dot{s}_k(t)} + \frac{\tau^2}{2!}\,\frac{d\dddot{s}_k(t)}{d\dot{s}_k(t)} - \cdots\cdots \tag{4.53}$$

Hence, a suitable numerical method for evaluating $\dfrac{d\ddot{s}_k(t)}{d\dot{s}_k(t)}$, etc. must be evolved in order to obtain the value of $\dfrac{\partial \dot{s}_k(t-\tau)}{\partial \dot{s}_k(t)}$. Then, the scheduling equation for the k^{th} hydro-plant can be solved numerically.

CHAPTER V

APPLICATION

5-1 The Problem Considered

The system considered here consists of one thermal and two hydro-electric plants. It is required to operate this system to meet a given load demand in the most economical manner. The characteristics of the hydro-plants and the cost function for the system are given in Appendix 1. The system is shown schematically in Fig. 5.1 The optimization interval is 24 hours. The two hydro-plants are located on the same stream. The effect of variation of head upon the hydro-plant characteristics will be taken into consideration.

The problem is solved under the following assumptions:

1. The time taken by the water to flow from the upstream plant to the downstream plant is assumed to be zero.
2. During the optimization period, the natural inflows to the reservoirs of the two plants are assumed to be zero.
3. The tail race elevations and the head losses in conduits are assumed to be constant.
4. The transmission losses in the system are negligible.

System Studied in Chapter 5

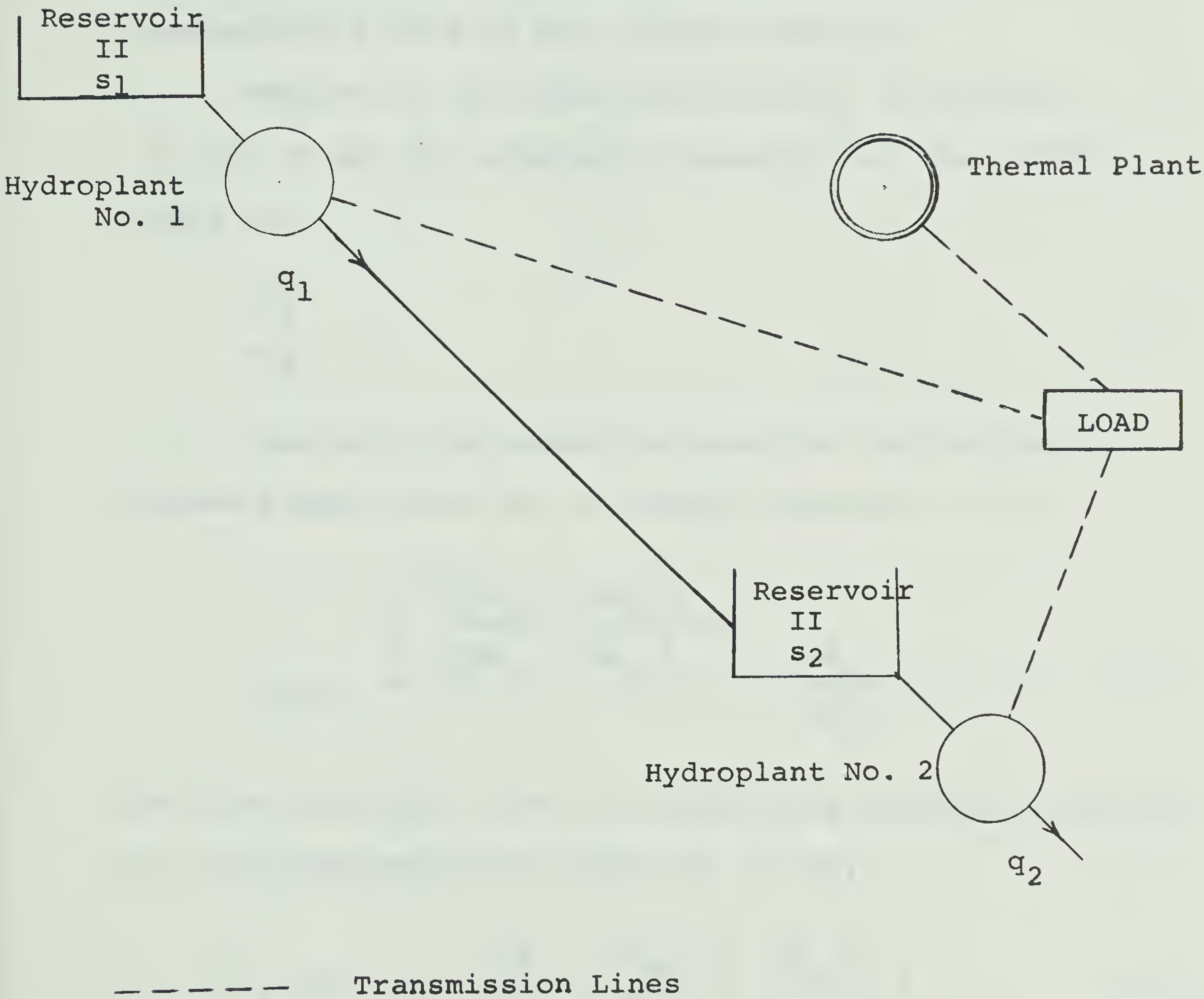

FIGURE 5-1

5-2 Method of Solution

The above-mentioned problem is solved by using the scheduling equations developed in section 4-4. The equations are simplified in accordance with the assumptions listed in the previous section.

Neglecting the terms containing P_L in equation (4.10), we get the scheduling equation for the thermal plant as,

$$\frac{\partial C_T}{\partial P_T} = \lambda \tag{5.1}$$

Similarly the scheduling equation for the down-stream plant (plant no. 2) becomes (equation 4.24)):

$$- \gamma_{(2)0} \cdot e^{\int_0^t \left[\frac{\partial P_{H2}}{\partial s_2} / \frac{\partial P_{H2}}{\partial \dot{s}_2}\right] dt} \cdot \frac{\partial \dot{s}_2}{\partial P_{H2}} = \lambda \tag{5.2}$$

and from equation (4.36), we obtain the scheduling equation for the upstream plant (plant no. 1) as,

$$\left[\gamma_1(t) - \gamma_2(t) \frac{\partial \dot{s}_2}{\partial P_{H2}} \cdot \frac{\partial P_{H2}}{\partial \dot{s}_1}\right] \left[- \frac{\partial \dot{s}_1}{\partial P_{H2}}\right] = \lambda \tag{5.3}$$

where

$$\gamma_1(t) = \gamma_{(1)0} \cdot e^{\displaystyle\int_0^t \frac{\frac{\partial P_{H1}}{\partial s_1}}{\frac{\partial P_{H1}}{\partial \dot{s}_1} + \frac{\partial P_{H2}}{\partial \dot{s}_1}} dt} \tag{5.4}$$

and

$$\gamma_2(t) = \gamma_{(2)0} \cdot e^{\displaystyle\int_0^t \frac{\frac{\partial P_{H2}}{\partial s_2}}{\frac{\partial P_{H2}}{\partial \dot{s}_2}} dt} \tag{5.5}$$

Equation (5.3) can be further simplified by taking into account the fact that $\tau = 0$. When $\tau = 0$, equation (2.9) becomes,

$$P_{H2} = P_{H2}\,(s_2, (\dot{s}_1 + \dot{s}_2), t) \tag{5.6}$$

By taking the partial derivatives of both the sides it is found that

$$\frac{\partial P_{H2}}{\partial \dot{s}_1} = \frac{\partial P_{H2}}{\partial \dot{s}_2} \tag{5.7}$$

Also, we know that [14],

$$\frac{\partial \dot{s}_2}{\partial P_{H2}} = \frac{1}{\frac{\partial P_{H2}}{\partial \dot{s}_2}} \tag{5.8}$$

therefore

$$\frac{\partial \dot{s}_2}{\partial P_{H2}} \cdot \frac{\partial P_{H2}}{\partial \dot{s}_1} = 1 \qquad (5.9)$$

Hence, equation (5.3) can be rewritten as,

$$\left[\gamma_1(t) - \gamma_2(t)\right] \left(-\frac{\partial \dot{s}_1}{\partial P_{H2}}\right) = \lambda \qquad (5.10)$$

Equations (5.2) and (5.10) can be rewritten as,

$$\frac{1}{\lambda} \cdot \gamma_2(t) = -\frac{\partial P_{H2}}{\partial \dot{s}_2} \qquad (5.11)$$

and

$$\frac{1}{\lambda}\left[\gamma_1(t) - \gamma_2(t)\right] = -\frac{\partial P_{H1}}{\partial \dot{s}_1} \qquad (5.12)$$

respectively. $\gamma_1(t)$ and $\gamma_2(t)$ are given by equations (5.4) and (5.5) respectively.

Now, from the given system data (Appendix 1)

$$C_T = 0.012\, P_T^2 + 4.0 P_T + 1.0 \qquad (5.13)$$

Hence, evaluating the partial derivative of C_T with respect to P_T and substituting into equation (5.1), we get

$$0.024\, P_T + 4.0 = \lambda \qquad (5.14)$$

or

$$P_T = \frac{\lambda - 4.0}{0.024} \qquad (5.15)$$

Equation (5.15) represents the scheduling equation for the thermal plant.

Also, from the system data (Appendix 1) for the downstream plant (plant No. 2), we have,

$$P_{H2} = 0.076\ q_2\ (h_2 - 5 - 1.5\ q_2) \tag{5.16}$$

Now, since the natural inflows are zero equation (2.5) becomes

$$q_2 = -\dot{s}_1 - \dot{s}_2 \tag{5.17}$$

Also

$$h_2 = \frac{s_2}{A_2} \tag{5.18}$$

Substituting for q_2 and h_2 in equation (5.11), we get

$$P_{H2} = 0.076\ (-\dot{s}_1 - \dot{s}_2) \left[\frac{s_2}{A_2} - 5 - 1.5\ (-\dot{s}_1 - \dot{s}_2)\right] \tag{5.19}$$

Hence,

$$\frac{\partial P_{H2}}{\partial \dot{s}_1} = \frac{\partial P_{H2}}{\partial \dot{s}_2} \tag{5.20}$$

$$= -0.076 \left[\frac{s_2}{A_2} - 5 - 3.0\ (-\dot{s}_1 - \dot{s}_2)\right]$$

$$= -0.076\ (h_2 - 5 - 3.0\ q_2) \tag{5.21}$$

and

$$\frac{\partial P_{H2}}{\partial s_2} = 0.076 \; \frac{q_2}{A_2} \tag{5.22}$$

Using equations (5.21 and (5.22), equation (5.2) for the downstream plant can be written as,

$$\frac{\gamma_2(t)}{\lambda} = 0.076 \; (h_2 - 5 - 3.0 \; q_2) \tag{5.23}$$

and, from equation (5.5)

$$\gamma_2(t) = \gamma_{(2)0} \, . e^{\int_0^t \frac{0.076 \quad q_2}{-A_2 . 0.076 (h_2 - 5.0 - 3.0 \; q_2)} dt}$$

$$= \gamma_{(2)0} . e^{\int_0^t \frac{- \quad q_2}{A_2 . (h_2 - 5.0 - 3.0 \; q_2)} dt} \tag{5.24}$$

Thus,

$$Z_2(t) = - \frac{q_2}{A_2 (h_2 - 5.0 - 3.0 \; q_2)} \tag{5.25}$$

Now, by solving equation (5.23) for q_2, we get,

$$q_2 = \frac{1}{3} \left[\frac{\gamma_2}{0.076 . \lambda} - h_2 + 5.0 \right] \tag{5.26}$$

From the system data (Appendix 1) the expression for the output of the upstream plant (plant No. 1) is,

$$P_{H1} = 0.06486.q_1(h_1 - 20 + 38.107\ q_1 - 2.863\ q_1^{\ 2}) \tag{5.27}$$

Since, the natural inflows are assumed to be zero,

$$q_1 = -\dot{s}_1 \tag{5.28}$$

Also

$$h_1 = \frac{s_1}{A_1} \tag{5.29}$$

Substituting for q_1 and h_1 in equation (5.27) and taking the partial derivatives with respect to s_1 and $\dot{s}_1$, we get

$$\frac{\partial P_{H1}}{\partial s_1} = \frac{0.06486\ (-\dot{s}_1)}{A_1}$$

$$= \frac{0.06486}{A_1}\ q_1 \tag{5.30}$$

and

$$\frac{\partial P_{H1}}{\partial \dot{s}_1} = -0.06486(h_1 - 20 + 76.214(-\dot{s}_1) - 8.589(-\dot{s}_1)^2)$$

$$= -0.06486(h_1 - 20 + 76.214\ q_1 - 8.589 q_1^{\ 2}) \tag{5.31}$$

Substituting for $\dfrac{\partial P_{H1}}{\partial \dot{s}_1}$ in equation (5.12), we get

$$\frac{1}{\lambda}\left[\gamma_1 - \gamma_2\right] = +\ 0.06486(h_1 - 20 + 76.214\ q_1 - 8.589\ q_1^{\ 2}) \quad (5.32)$$

Rearranging the terms and solving for q_1, we get

$$q_1 = \frac{76.214 \pm \sqrt{(76.214)^2 - 34.356\ C_1}}{17.178} \quad (5.33)$$

where

$$C_1 = \frac{\gamma_1 - \gamma_2}{\lambda \cdot 0.06486} - h_1 + 20 \quad (5.34)$$

Equation (5.6) can be written as

$$\gamma_1(t) = \gamma_{(1)0}\ e^{\int_0^t Z_1(t)\ dt} \quad (5.35)$$

where

$$Z_1(t) = \frac{\dfrac{\partial P_{H1}}{\partial s_1}}{\dfrac{\partial P_{H1}}{\partial \dot{s}_1} + \dfrac{\partial P_{H2}}{\partial \dot{s}_1}} \quad (5.36)$$

Substituting for $\frac{\partial P_{H1}}{\partial \dot{s}_1}$, $\frac{\partial P_{H1}}{\partial s_1}$ and $\frac{\partial P_{H2}}{\partial \dot{s}_1}$ from equations (5.30), (5.31) and (5.21) in equation (5.36), we get

$$Z_1(t) = \frac{-\ 0.06486\ .\ q_1}{A_1\left[0.06486(h_1 - 20 + 76.214\ q_1\ \ -8.859\ q_1^{\ 2}) + 0.076(h_2 - 5 - 3.0\ q_2)\right]} \quad (5.37)$$

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5-3 Computing Technique

The optimization interval is divided into 24 sub-intervals, each of one hour duration. The required generation schedule is obtained by solving equations (5.15), (5.26) and (5.33). For known values of λ, $\gamma_{(1)0}$ and $\gamma_{(2)0}$, these equations can be solved for P_T, q_1 and q_2. By substituting for q_1 and q_2 in equations (5.27) and (5.16) the corresponding values of P_{H1} and P_{H2} can be obtained. The values of λ and $\gamma_{(1)0}$ and $\gamma_{(2)0}$ are found by using the constraints of the problem, as explained in the following paragraphs.

λ appears in the three scheduling equations due to the point constraint that the load demand is to be met at each hourly interval. That is, we require

$$P_D = P_{H1} + P_{H2} + P_T \tag{5.38}$$

This relationship must be satisfied at all times.

The values of $\gamma_{(1)0}$ and $\gamma_{(2)0}$ are chosen such that the total amount of water used by each plant over the optimization interval is equal to the limited specified quantity. That is,

$$\int_0^T q_j \, dt = Q_j \tag{5.39}$$

$$j = 1, 2$$

The purpose of writing the computer program is not only to solve the three scheduling equations, but also to compute the values for λ and $\gamma_{(1)0}$ and $\gamma_{(2)0}$. The general layout of the program is given by the flow-chart given on page 62 of this thesis.

The scheduling equations (5.15), (5.26) and (5.33) can be solved for known values of λ, $\gamma_{(1)0}$ and $\gamma_{(2)0}$. The constraint given by equation (5.38) must be satisfied at all instants during the optimization interval. For this reason a λ loop appears within the γ loop of the program. The values of $\gamma_{(1)0}$ and $\gamma_{(2)0}$ can be corrected only when a trial schedule for 24 hours has been calculated.

Hence, to start with, some suitable values for $\gamma_{(1)0}$ and $\gamma_{(2)0}$ are assumed and the scheduling equations are subsequently solved for the 24 intervals. At each of these intervals new value value for λ is to be obtained. This is done, by assuming a suitable starting value for λ and solving the scheduling equations. Now, if the constraint given by equation (5.38) is not satisfied, the value of λ is increased by a small increment and the above process is repeated until the constraint is satisfied. Once this is done, the computer leaves the loop and goes on to calculate new values for γ_1, γ_2, and h_1 and h_2 in order to compute the generation schedules for the next hour. Glimn and Kirchmayer [14] have suggested keeping $Z_1(t)$ and $Z_2(t)$ constant during all of the

FLOW DIAGRAM

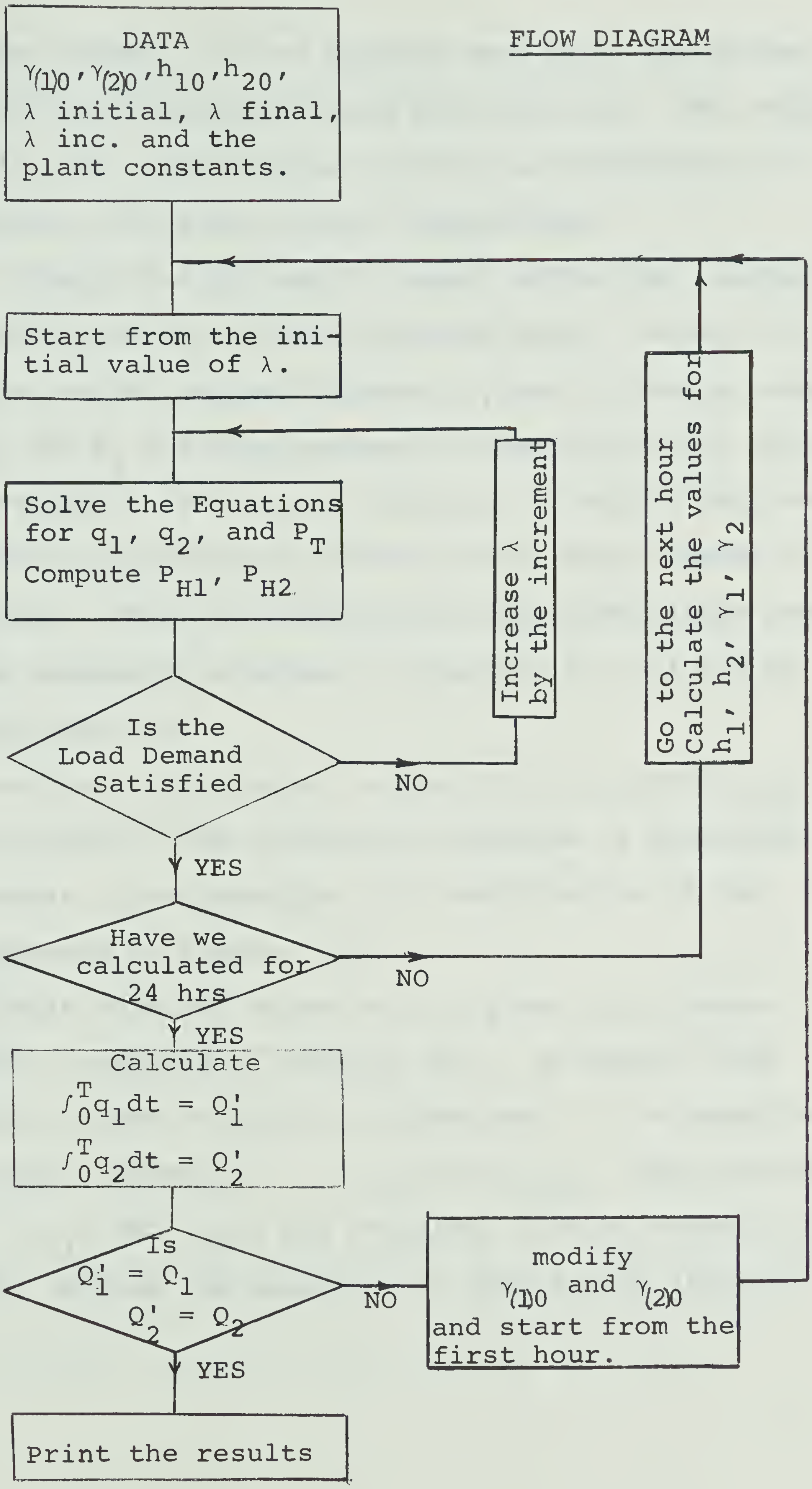

FIGURE 5-2

optimization period. In the program used here, new values of $Z_1(t)$ and $Z_2(t)$ are calculated for each hour. The values for γ_1 and γ_2 for a particular interval are evaluated by using equation (5.35) and (5.24) respectively.

The integration is done by simply adding the values of $Z_1(t)$ and $Z_2(t)$ for all the previous hours. Also, it is assumed that during any sub-interval γ_1 and γ_2 remain constant. q_1, q_2, h_1 and h_2 are also assumed to remain constant during the sub-intervals. For better accuracy, it may be desirable to divide the optimization period into a larger number of sub-intervals. Thus, by following the above-mentioned procedure, the generation schedule is prepared for all the 24 hourly sub-intervals.

To arrive at the correct values for $\gamma_{(1)0}$ and $\gamma_{(2)0}$ a method similar to the relaxation technique is developed in this thesis. The technique is a modification of the method suggested by Dandeno [22].

Suitable starting values of $\gamma_{(1)0}$ and $\gamma_{(2)0}$ should be available, computed or guessed [22]. By making rough calculations, based on logical assumptions, it is possible to find rough values of λ, $\gamma_{(1)0}$ and $\gamma_{(2)0}$. Once suitable values of $\gamma_{(1)0}$ and $\gamma_{(2)0}$ are obtained, a trial schedule is prepared by solving the equations at each hourly interval.

Let the amount of water used by the two plants, over the optimization interval, be Q_1' and Q_2' respectively. The values of $\gamma_{(1)0}$ and $\gamma_{(2)0}$ are to be corrected, so that the total amount of water used is Q_1 and Q_2. To do this two more schedules are prepared by changing the values of $(\gamma_{(1)0} - \gamma_{(2)0})$ and $\gamma_{(2)0}$ in turn. Hence, first $\gamma_{(1)0}$ is changed by a small increment $\Delta\gamma_1$, while $\gamma_{(2)0}$ is kept constant; and then both $\gamma_{(1)0}$ and $\gamma_{(2)0}$ are changed by $\Delta\gamma_2$ so that $(\gamma_{(1)0} - \gamma_{(2)0})$ remains constant. Let the water required by the two plants in the two cases be Q_1'' and Q_2''; and Q_1''' and Q_2'''.

Let us define,

$$R_1 = Q_1 - Q_1' \tag{5.40}$$

$$R_2 = Q_2 - Q_2' \tag{5.41}$$

$$\Delta Q_1' = Q_1' - Q_1'' \tag{5.42}$$

$$\Delta Q_2' = Q_2' - Q_2'' \tag{5.43}$$

$$\Delta Q_1'' = Q_1' - Q_1''' \tag{5.44}$$

$$\Delta Q_2'' = Q_2' - Q_2''' \tag{5.45}$$

Now, let the required correction in the values of $\gamma_{(1)0}$ and $\gamma_{(2)0}$ be $\delta\gamma_1$ and $\delta\gamma_2$. The values of $\delta\gamma_1$ and $\delta\gamma_2$ are obtained by solving the following equations:

$$(\delta\gamma_1 - \delta\gamma_2) \frac{\Delta Q_1'}{\Delta\gamma_1} + \delta\gamma_2 \frac{\Delta Q_1''}{\Delta\gamma_2} = R_1 \tag{5.46}$$

$$(\delta\gamma_1 - \delta\gamma_2) \frac{\Delta Q_2'}{\Delta\gamma_1} + \delta\gamma_2 \frac{\Delta Q_2''}{\Delta\gamma_2} = R_2 \tag{5.47}$$

The corrected values of $\gamma_{(1)0}$ and $\gamma_{(2)0}$ are given by,

$$\gamma_{(1)0} \text{ (corrected)} = \gamma_{(1)0} + \delta\gamma_1 \tag{5.48}$$

$$\gamma_{(2)0} \text{ (corrected)} = \gamma_{(2)0} + \delta\gamma_2 \tag{5.49}$$

A new schedule is now prepared by using the new values for $\gamma_{(1)0}$ and $\gamma_{(2)0}$. The process may have to be repeated a number of times, till the constraint given by equation (5.39) is satisfied. The number of such iterations, which is required, depends upon the extent to which the constraint (given by equation (5.39)) has to be satisfied and the starting values of $\gamma_{(1)0}$ and $\gamma_{(2)0}$. This procedure is found to work quite satisfactorily in all the cases considered in the following paragraphs.

The complete computer program is given in Appendix II. The computer results for the problem are given in Appendix III. Table I on page 66 gives the initial and the corrected values for $\gamma_{(1)0}$ and $\gamma_{(2)0}$ and the corresponding total discharges of the two hydro-plants.

TABLE I

Calculation of the Value of $\gamma_{(1)0}$ and $\gamma_{(2)0}$

	$\gamma_{(1)0}$	$\gamma_{(2)0}$	Total Water Used by Hydro-Plant No. 1 in 24 hrs.	Total Water Used by Hydro-Plant No. 2 in 24 hrs.
Initial Values	225.000	65.000	139.76	131.17
1st Corrected Values	241.651	67.995	123.62	153.18
2nd Corrected Values	241.259	68.048	124.98	150.05
3rd Corrected Values	241.258	68.050	125.00	150.00

Amount of water available for use by plant No. 1 = 125 K.S.F.-hr.

Amount of water available for use by plant No. 2 = 150 K.S.F.-hr.

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5.4 Solution by Alternative Methods and Comparison of Results

To test the effectiveness of the method developed in this thesis, the problem discussed in section 5-1 is solved by using the following methods as well.

Alternative Method No. 1:

Constant γ Method:

This method has been developed by Kirchmayer [18]. It does not take into account the effect of variation of head upon hydro-plant characteristics. Neglecting transmission loss terms, the scheduling equations become, for the thermal plant:

$$\frac{\partial C_T}{\partial P_T} = \lambda \tag{5.50}$$

and for the hydro-plant number 1:

$$(\gamma_1 - \gamma_2)\frac{\partial q_2}{\partial P_{H2}} = \lambda \tag{5.51}$$

and for hydro-plant number 2:

$$\gamma_2 \frac{\partial q_2}{\partial P_{H2}} = \lambda \tag{5.52}$$

Both γ_1 and γ_2 remain constant during the optimization interval. The problem is solved under the same assumptions and on the same lines as those used in section 5-3.

Alternative Method No. 2:

One possible method for operating the hydro-plants is to keep their discharge constant at an average value. The total output of the two hydro-plants is computed and the deficit is generated by the thermal plant. The cost of operating the system in this way is calculated.

Alternative Method No. 3:

The problem stated in section 3-1 is modified by assuming that the two hydro-plants are located on separate streams. The scheduling equations for the thermal plant and the downstream hydro-plant are the same as equations (5.1) and (5.2) respectively. The scheduling equation for the upstream plant becomes: (from equation (3.47))

$$\gamma_{(1)0} \cdot e^{\int_0^t \frac{\partial P_{H1}}{\partial s_1} / \frac{\partial P_{H1}}{\partial \dot{s}_1} dt} = - \lambda \frac{\partial P_{H1}}{\partial \dot{s}_1} \qquad (5.53)$$

The three equations are solved under the same assumptions and by using the same method for numerical solution as those discussed in section 5-3.

Comparison of Results

The cost of operating the system according to the generation schedule obtained by using the method developed in this thesis; and by using the alternative methods described in this section are given in Table II on page 70 of this thesis. The total amount of energy generated by the thermal plant of the system, over the optimization interval, is also given in that table.

It is found that the cost of operating the system is a minimum when we use the scheduling equations derived in this thesis.

TABLE II

Comparison of Results

Method of Solution	Cost of Operating the System for 24 hours in $	Total Mw-hr Generated by Thermal Plant Over 24 hours
1. Using the scheduling equations developed in this thesis	3209	729.69
2. Constant γ Method	3214	731.16
3. Discharge of the two plant kept constant at an average value	3225	733.14
4. The problem is modified by assuming that the hydro-plants of the system are on separate streams. The problem is solved by using the scheduling equations developed in section 3-5.	3377	765.18

CHAPTER VI
CONCLUSIONS AND REMARKS

General scheduling equations have been developed for the thermal and variable head hydro-electric plants of an interconnected power system. The scheduling equations for the hydro-plants have been obtained by using the Euler equation in a "modified" form, developed in this thesis. The transmission line losses have been taken into account while deriving the equations.

The scheduling equations for the hydro-plants on separate streams are shown to be equivalent to those by Glimn and Kirchmayer [14].

While deriving the scheduling equations for the common-flow plants, (section 4-4) it has been assumed for simplicity, that only two hydro-plants are located on the same stream. The discussion, however, is quite general in character and can be extended to cases where more than two plants are on the same stream. The time (τ) taken by water to flow from the upstream plant to the downstream plant has been taken into consideration while formulating the problem and deriving the scheduling equations. But, much remains to be done before the equations can be successfully used. Suitable numerical technique is to be found for evaluating $\frac{d\ddot{s}_k(t)}{d\dot{s}_k(t)}$, $\frac{d\dddot{s}_k(t)}{d\dot{s}_k(t)}$, etc. in order

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to find the value of $\dfrac{\partial \dot{s}_k(t-\tau)}{\partial \dot{s}_k}$ (discussed in section 4-5). Also while expanding $\dot{s}_k(t-\tau)$ by Taylor's series, it has been assumed that τ is constant. However, strictly speaking, τ depends upon the discharge of the upstream plant. This further complicates the problem of evaluating $\dfrac{\partial P_{Hk+1}}{\partial \dot{s}_k}$. Obviously, a careful analysis of this aspect of the problem is highly desirable.

The scheduling equations obtained in this thesis have been applied to a small system consisting of one thermal and two hydro-electric power plants. The two hydro-plants are located on the same stream. τ and the transmission line losses are assumed to be zero.

The same problem has been solved under the same assumptions and simplification by using a few other (approximate) methods described in section 5-4. It is found that the most economical generation schedule is obtained by using the scheduling equations developed in this thesis.

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APPENDIX 1

System Characteristics for the Problem Discussed in Chapter 5

The system consists of one thermal and two hydro-plants. The cost function for the system and the characteristics of the hydro-plants are taken from reference [15]. The expression for the output of the hydro-plant no. 2 was modified so that the incremental water usage curve ($\frac{\partial q_j}{\partial P_{Hj}}$ vs. P_{Hj}) has a positive slope.

The subscripts 1 and 2 refer to the upstream and downstream plant respectively.

1. Cost function

$$C_T = 0.024\ P_T^{\,2} + 4.0\ P_T + 1.0$$

2. Plant discharges

$$q_1 = -\dot{s}_1$$

When the hydro-plants are on the same river

$$q_2 = -\dot{s}_1 - \dot{s}_2$$

and when the hydro-plants are on separate streams

$$q_2 = -\dot{s}_2$$

3. Effective Head

The tailrace elevation and the head losses in the conduits are taken to be constant, so that h_1 and h_2 can be represented by the following relationships:

$$h_1 = h_1(0) - \frac{\int_0^t q_1 \, dt}{A_1}$$

When the hydro-plants are on the same stream

$$h_2 = h_2(0) - \frac{\int_0^t (q_2 - q_1) \, dt}{A_2}$$

and when the hydro-plants are on separate streams

$$h_2 = h_2(0) - \frac{\int_0^t q_2 \, dt}{A_2}$$

where, A_1 and A_2 are the surface areas of the reservoirs of the two hydro-plants. These are assumed to be constant within the operating range, and their values are,

$$A_1 = 25 \times \frac{3600 \times 10^3}{43560} = 2310 \text{ Acres}$$

and

$$A_2 = 20 \times \frac{3600 \times 10^3}{43560} = 1651 \text{ Acres}$$

4. Power Generated

$$P_{H1} = 0.06489q_1(h_1-20+38.107q_1-2.863q_1^2)$$

$$P_{H2} = 0.076(h_2-5-1.5\ q_2)$$

5. Minimum discharges

$$q_1 = 0$$

$$q_2 = 0$$

6. System Load Demand

The load demand is assumed to remain constant at 300 Mw. throughout the optimization interval of 24 hours.

7. Total water available for use

$$Q_1 = \int_0^T q_1\,dt = 125 \text{ k.s.f. hr.}$$

$$Q_2 = \int_0^T q_2\,dt = 150 \text{ k.s.f. hr.}$$

where

$T = 24$ hours.

APPENDIX 2

Computer Program

```
C     HYDRO-THERMAL DISPATCH  COMMON FLOW PLANTS
      DIMENSIONXG1(30),XG2(30),PD(30),H1(30),H2(30),D1(30),D2(30),Z1(30)
     X,Z2(30),PT(30),PH1(30),PH2(30),P(30),ER(30),Z11(30),Z01(30),Z02(30
     X),D11(30),C1(30),C2(30),Q1(30),Q2(30),G(30),D12(30),D21(30),D22(30
     X),GC(30),S1(10),S2(10)
      READ(5,1)SER,H10,H20,XK1,XK2,IL,JK,KL
    1 FORMAT(5F5.1,3I7)
      READ(5,11)Q10,Q20,DG1,DG2,XG11,XG21,ERQ,K
   11 FORMAT(7F8.3,1I2)
      READ(5,2)(PD(I),I=1,24)
    2 FORMAT(12F5.1/12F5.1)
      XG20=XG21
      XG10=XG11
      J=1
      L=1
   72 WRITE(6,100)
  100 FORMAT(1H1)
      WRITE(6,101)
  101 FORMAT(////////)
      WRITE(6,71)
   71 FORMAT(3HHHR,4X,1HL,5X,2HG1,7X,2HG2,5X,2HH1,6X,2HH2,5X,2HD1,4X,2HD
     X2,5X,3HPH1,5X,3HPH2,5X,2HPT,4X,2HPD,5X,3HERR,5X,2HZ1,7X,2HZ2,7X,2H
     XQ1,6X,2HQ2)
      WRITE(6,102)
  102 FORMAT(/)
      CD=0.
      EF=0.
      IA=1
      G1=XG10
      G2=XG20
      XH1=H10
      XH2=H20
      DO29LA=IL,JK,KL
      XL=LA
      A=XL/100000.
      XC1=(((G1-G2)/(0.06486*A))-XH1+20.)
      XD12=(76.214*76.214-34.356*XC1)
      IF(XD12)29,30,30
   30 XD11=SQRT(XD12)
      XD1=(76.214+XD11)/17.178
      XPH1=0.06486*XD1*(XH1+38.107*XD1-2.863*XD1*XD1-20.)
      XD2=(XH2-5.-(G2/(A*0.076)))/3.0
      IF(XD2)36,37,37
   36 XD2=0.0
   37 XPH2=0.076*XD2*(XH2-5.-1.5*XD2)
      XPT=(A-4.0)/0.024
      XP=XPH1+XPH2+XPT
      XER=PD(1)-XP
      AAR=ABS(XER)
      IF(SER-AAR)9,10,10
```

Contd..

```
 9 XD1=(76.214-XD11)/17.178
   XPH1=0.06486*XD1*(XH1+38.107*XD1-2.863*XD1*XD1-20.)
   XP=XPH1+XPH1+XPT
   XER=PD(1)-XP
   AAR=ABS(XER)
   IF(SER-AAR)29,10,10
29 CONTINUE
10 WRITE(6,4)IA,A,G1,G2,XH1,XH2,XD1,XD2,XPH1,XPH2,XPT,PD(1),XER,CD,EF
  X,XD1,XD2
 4 FORMAT(I3,1F6.3,4F8.3,2F6.3,3F8.3,1F6.1,3F8.4,2F8.2)
   Q1(1)=XD1
   Q2(2)=XD2
   H1(1)=XH1
   H2(2)=XH2
   D1(1)=XD1
   D2(2)=XD2
   Z01(1)=0
   Z02(2)=0
   GC(1)=0.012*XPT**2+4.*XPT+1
   DO7I=2,24
   H1(I)=H1(I-1)-D1(I-1)/XK1
   H2(I)=H2(I-1)-D2(I-1)/XK2+XD1/XK2
   Z2(I)=D2(I-1)/((H2(I-1)-5.-3.*D2(I-1))*XK2)
   Z02(I)=Z02(I-1)+Z2(I)
   Z11(I)=H1(I-1)-20.+76.214*D1(I-1)-8.589*D1(I-1)**2.+(0.076/0.06486
  X)*(H2(I-1)-5.-3.*D2(I-1))
   Z1(I)=D1(I-1)/(Z11(I)*XK1)
   Z01(I)=Z01)I-1)-Z1(I)
   ZG2(I)=XG20/EXP(Z02(I))
   ZG1(I)=XG20/EXP(Z02(I))
   DO3LL=IL,JK,KL
   XL=LL
   XL=XL/100000.
   PT(I)=(XL-4.)/0.024
   C1(I)=((XG1(I)-XG2(I))/(0.06486*XL))-H1(I)+20.
   D12(I)=(76.214*76.214-34.356*C1(I)0
   IF(D12(I))3,34,34
34 D11(I)=SQRT(D12(I))
   D1(I)=(76.214+D11(I))/17.178
   PH1(I)=0.06486*D1(I)*(H1(I)+38.107*D1(I)-2.863*D1(I)*D1(I)-20.)
   C2(I)=((XG2(I)/0.076*XL))-H2(I)+5.)
   D22(I)=C2(I)/3.0
   D2(I)=D22(I)*(-1)
   IF(D2(I))45,46,46
45 D2(I)=0
46 PH2(I)=0.076*D2(I)*(H2(I)-5.-1.5*D2(I))
   P(I)=PH1(I)+PH2(I)+PT(I)
   ER(I)=PD(I)-P(I)
   AER=ABS(ER(I))
   IF(SER-AER)33,19,19
33 D1(I)=(76.214-D11(I))/17.178
   PH1(I)=0.06486*D2(I)*(H1(I)+38.107*D1(I)-2.863*D1(I)*D1(I)-20.)
   P(I)=PH1(I)+PH2(I)+PT(I)
   ER(I)=PD(I)-P(I)
```

Contd.

```
      AER=ABS(ER(I))
      IF(SER-AER)3,19,19
    3 CONTINUE
   19 Q1(I)=Q1(I-1)+D1(I)
      Q2(I)=Q2(I-1)+D2(I)
      G(I)=.012*PT(I)**2+4.0*PT(I)+1.
      GC(I)=GC(I-1)+G(I)
      WRITE(6,4)I,XL,XG1(I),XG2(I),H1(I),H2(I),D1(I),D2(I),PH1(I),PH2(I)
     X,PT(I),PD(I),ER(I),Z1(I),Z2(I),Q1(I),Q2(I)
    7 CONTINUE
      WRITE(6,103)
  103 FORMAT(////)
      WRITE(6,60)
   60 FORMAT(20X,4HCOST)
      COST=GC(24)
      WRITE(6,43)COST
   43 FORMAT(20X,1F10.3)
      S1(J)=Q1(24)
      S2(J)=Q2(24)
      R2=Q20-S2(I)
      R1=010-S1(I)
      AR1=ABS(R1)
      AR2=ABS(R2)
      IF(ERQ-AR1)90,91,91
   91 IF(ERQ-AR2)90,77,77
   90 IF(L-K)76,76,77
   76 IF(J-2)73,74,75
   73 XG10=XG11+DG1
      XG20=XG21
      J=J+1
      GO TO 72
   74 XG10=XG11+DG2
      XG20=XG21+DG2
      J=J+1
      GO TO 72
   75 L=L+1
      X1=(S1(2)-S1(1))/DG1
      X2=(S1(3)-S1(1))/DG2
      Y1=(S2(2)-S2(1))/DG1
      Y2=(S2(3)-S2(1))/DG2
      X=(R1*Y2-R2*X2)/(X1*Y2 Y1*X2)
      Y=(R1-X*X1)/X2
      XG20=XG21+Y
      XG10=XG11+X+Y
      J=1
      XG11=XG10
      XG21=XG20
      GO TO 72
   77 STOP
      END
```

APPENDIX 3

COMPUTER RESULTS

Explanation of the Symbols in Computer Results

Symbols in Computer Results		Symbols used in the remainder of this thesis (for explanation, see page viii and ix.)
HR	=	Hour
L	=	λ
G1	=	γ_1
G2	=	γ_2
H1	=	$h_1(t)$
H2	=	$h_2(t)$
D1	=	$q_1(t)$
D2	=	$q_2(t)$
PH1	=	$P_{H1}(t)$
PH2	=	$P_{H2}(t)$
PT	=	$P_T(t)$
PD	=	$P_D(t)$
ERR	=	$P_D - P_{H1} - P_{H2} - P_T$
Z1	=	$-Z_1(t)$
Z2	=	$-Z_2(t)$
Q1	=	$\int_0^t q_1 \, dt$
Q2	=	$\int_0^t q_2 \, dt$
G(1)0	=	$\gamma(1)0$
G(2)0	=	$\gamma(2)0$

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Initial Values

G(1)Q = 225.000
G(2)U = 65.000

HR	L	G1	G2	H1	H2	D1	D2	PH1	PH2	PT	PJ	ERR	Z1	Z2	Q1	Q2
1	4.479	225.000	65.000	425.000	210.000	6.084	4.683	209.490	70.467	19.958	300.0	0.0842	0.0	0.0	6.09	4.63
2	4.476	224.929	64.920	424.757	210.070	6.063	4.747	208.635	71.419	19.848	300.0	0.0996	0.0003	0.0012	12.15	9.43
3	4.474	224.859	64.840	424.514	210.136	6.042	4.813	207.779	72.388	19.743	300.0	0.0894	0.0003	0.0012	18.19	14.24
4	4.471	224.789	64.758	424.272	210.197	6.020	4.878	206.916	73.359	19.640	300.0	0.0847	0.0003	0.0013	24.21	19.12
5	4.469	224.719	64.675	424.031	210.254	5.998	4.944	206.044	74.331	19.538	300.0	0.0867	0.0003	0.0013	30.21	24.06
6	4.467	224.649	64.591	423.791	210.307	5.976	5.009	205.163	75.303	19.438	300.0	0.0959	0.0003	0.0013	36.18	29.07
7	4.464	224.580	64.506	423.552	210.355	5.954	5.076	204.278	76.284	19.342	300.0	0.0964	0.0003	0.0013	42.14	34.15
8	4.462	224.511	64.420	423.314	210.399	5.932	5.143	203.389	77.275	19.248	300.0	0.0869	0.0003	0.0013	48.07	39.27
9	4.460	224.443	64.333	423.076	210.439	5.910	5.211	202.490	78.257	19.157	300.0	0.0874	0.0003	0.0014	53.98	44.50
10	4.458	224.374	64.245	422.840	210.474	5.887	5.279	201.580	79.255	19.067	300.0	0.0959	0.0003	0.0014	59.87	49.78
11	4.456	224.306	64.155	422.604	210.504	5.864	5.347	200.664	80.259	18.980	300.0	0.0979	0.0003	0.0014	65.73	55.13
12	4.454	224.238	64.065	422.370	210.530	5.841	5.417	199.743	81.267	18.897	300.0	0.0940	0.0003	0.0014	71.57	60.55
13	4.452	224.171	63.973	422.136	210.551	5.817	5.486	198.810	82.276	18.815	300.0	0.0996	0.0003	0.0014	77.39	66.03
14	4.450	224.104	63.881	421.903	210.567	5.794	5.557	197.880	83.300	18.738	300.0	0.0823	0.0003	0.0015	83.19	71.59
15	4.448	224.037	63.787	421.671	210.579	5.770	5.626	196.927	84.317	18.662	300.0	0.0952	0.0003	0.0015	88.95	77.22
16	4.446	223.970	63.692	421.440	210.586	5.746	5.699	195.975	85.349	18.590	300.0	0.0859	0.0003	0.0015	94.70	82.92
17	4.444	223.904	63.595	421.210	210.589	5.721	5.771	195.010	86.380	18.520	300.0	0.0901	0.0003	0.0015	100.42	88.69
18	4.443	223.838	63.498	420.981	210.585	5.697	5.844	194.035	87.419	18.455	300.0	0.0925	0.0003	0.0015	106.11	94.53
19	4.441	223.772	63.399	420.755	210.579	5.672	5.916	193.053	88.465	18.390	300.0	0.0923	0.0003	0.0015	111.79	100.45
20	4.440	223.707	63.300	420.526	210.566	5.647	5.992	192.064	89.517	18.330	300.0	0.0896	0.0003	0.0015	117.43	106.44
21	4.439	223.642	63.199	420.300	210.549	5.621	6.067	191.064	90.576	18.275	300.0	0.0852	0.0003	0.0016	123.05	112.51
22	4.437	223.577	63.096	420.075	210.527	5.596	6.142	190.055	91.642	18.220	300.0	0.0833	0.0003	0.0016	128.65	118.65
23	4.436	223.513	62.993	419.851	210.500	5.569	6.218	189.028	92.706	18.163	300.0	0.0981	0.0003	0.0016	134.22	124.87
24	4.435	223.449	62.889	419.626	210.467	5.543	6.295	187.996	93.763	18.122	300.0	0.0991	0.0003	0.0017	139.76	131.17

COST
1943.786

G(1)0 = 225.000 + 0.1
G(2)0 = 65.000

KK	L	G1	G2	H1	H2	D1	D2	PH1	PH2	P1	P0	ERR	Z1	Z2	Q1	Q2
1	4.480	225.100	65.000	425.000	210.000	6.077	4.699	209.223	70.690	20.005	300.0	0.0840	0.0	0.0	6.08	4.70
2	4.477	225.029	64.920	424.757	210.069	6.055	4.703	208.357	71.643	19.895	300.0	0.0906	0.0003	0.0012	12.13	9.46
3	4.475	224.957	64.839	424.514	210.133	6.034	4.826	207.508	72.615	19.790	300.0	0.0867	0.0003	0.0012	18.17	14.27
4	4.472	224.889	64.757	424.275	210.194	6.012	4.894	206.641	73.584	19.687	300.0	0.0887	0.0003	0.0013	24.18	19.18
5	4.470	224.819	64.674	424.037	210.250	5.991	4.959	205.763	74.553	19.585	300.0	0.0946	0.0003	0.0013	30.17	24.14
6	4.468	224.750	64.590	423.797	210.301	5.969	5.025	204.383	75.532	19.487	300.0	0.0986	0.0003	0.0013	36.14	29.17
7	4.465	224.681	64.504	423.554	210.348	5.947	5.092	203.999	76.518	19.392	300.0	0.0906	0.0003	0.0013	42.08	34.26
8	4.463	224.612	64.416	423.310	210.391	5.924	5.160	203.105	77.507	19.298	300.0	0.0901	0.0003	0.0013	48.01	39.42
9	4.461	224.543	64.331	423.079	210.427	5.901	5.227	202.199	78.496	19.207	300.0	0.0984	0.0003	0.0014	53.91	44.65
10	4.459	224.475	64.242	422.845	210.463	5.879	5.296	201.290	79.494	19.116	300.0	0.0954	0.0003	0.0014	59.79	49.94
11	4.457	224.407	64.152	422.607	210.491	5.855	5.365	200.376	80.499	19.033	300.0	0.0916	0.0003	0.0014	65.64	55.31
12	4.455	224.339	64.062	422.375	210.517	5.832	5.434	199.449	81.505	18.950	300.0	0.0957	0.0003	0.0014	71.49	60.74
13	4.453	224.272	63.970	422.139	210.537	5.809	5.504	198.516	82.523	18.870	300.0	0.0942	0.0003	0.0014	77.29	66.25
14	4.451	224.205	63.877	421.907	210.552	5.785	5.574	197.579	83.541	18.793	300.0	0.0969	0.0003	0.0015	83.07	71.82
15	4.449	224.138	63.782	421.676	210.562	5.761	5.645	196.628	84.563	18.713	300.0	0.0911	0.0003	0.0015	88.83	77.46
16	4.448	224.071	63.687	421.445	210.568	5.737	5.717	195.670	85.593	18.647	300.0	0.0911	0.0003	0.0015	94.57	83.18
17	4.446	224.005	63.591	421.216	210.567	5.713	5.789	194.705	86.629	18.578	300.0	0.0884	0.0003	0.0015	100.28	88.97
18	4.444	223.939	63.493	420.987	210.565	5.688	5.863	193.732	87.675	18.515	300.0	0.0820	0.0003	0.0015	105.97	94.83
19	4.443	223.874	63.394	420.760	210.557	5.663	5.936	192.743	88.716	18.450	300.0	0.0916	0.0003	0.0016	111.63	100.77
20	4.441	223.808	63.294	420.533	210.543	5.638	6.011	191.755	89.775	18.392	300.0	0.0825	0.0003	0.0016	117.27	106.78
21	4.440	223.744	63.192	420.307	210.524	5.612	6.085	190.744	90.830	18.335	300.0	0.0908	0.0003	0.0016	122.88	112.87
22	4.438	223.679	63.090	420.083	210.501	5.587	6.161	189.727	91.892	18.282	300.0	0.0999	0.0003	0.0016	128.47	119.03
23	4.436	223.614	62.986	419.859	210.472	5.561	6.238	188.708	92.969	18.233	300.0	0.0996	0.0003	0.0016	134.03	125.26
24	4.437	223.550	62.881	419.637	210.436	5.534	6.315	187.676	94.052	18.188	300.0	0.0845	0.0003	0.0017	139.56	131.58

COST
1945.577

G(1)0= 225.000 + 0.1
G(2)0= 65.000 + 0.1

HR	L	G1	G2	H1	H2	D1	D2	PH1	PH2	PT	PD	ERR	Z1	Z2	Q1	Q2
1	4.483	225.100	65.100	425.000	210.000	6.097	4.636	210.033	69.773	20.195	300.0	0.0894	0.0	0.0	6.10	4.54
2	4.480	225.029	65.021	424.756	210.073	6.076	4.700	209.187	70.730	19.995	300.0	0.0886	0.0003	0.0012	12.18	9.34
3	4.477	224.958	64.941	424.513	210.142	6.057	4.764	208.333	71.688	19.887	300.0	0.0928	0.0003	0.0012	18.23	14.10
4	4.475	224.888	64.860	424.270	210.207	6.036	4.829	207.475	72.655	19.782	300.0	0.0876	0.0003	0.0012	24.27	18.93
5	4.472	224.818	64.778	424.029	210.267	6.014	4.894	206.610	73.624	19.678	300.0	0.0879	0.0003	0.0013	30.28	23.82
6	4.470	224.748	64.695	423.786	210.323	5.992	4.960	205.733	74.594	19.577	300.0	0.0959	0.0003	0.0013	36.28	28.78
7	4.467	224.679	64.611	423.548	210.375	5.970	5.026	204.855	75.574	19.478	300.0	0.0930	0.0003	0.0013	42.25	33.81
8	4.465	224.610	64.525	423.309	210.422	5.948	5.093	203.972	76.561	19.383	300.0	0.0835	0.0003	0.0013	48.20	38.90
9	4.463	224.541	64.439	423.071	210.465	5.926	5.160	203.073	77.542	19.288	300.0	0.0974	0.0003	0.0013	54.12	44.06
10	4.461	224.472	64.352	422.834	210.503	5.903	5.228	202.176	78.540	19.198	300.0	0.0659	0.0003	0.0014	60.03	49.23
11	4.459	224.404	64.265	422.598	210.537	5.881	5.297	201.267	79.538	19.110	300.0	0.0845	0.0003	0.0014	65.91	54.59
12	4.457	224.336	64.173	422.363	210.565	5.858	5.365	200.348	80.537	19.023	300.0	0.0923	0.0003	0.0014	71.76	59.95
13	4.455	224.268	64.083	422.126	210.590	5.834	5.434	199.423	81.543	18.940	300.0	0.0942	0.0003	0.0014	77.60	65.39
14	4.453	224.201	63.991	421.895	210.610	5.811	5.504	198.492	82.555	18.860	300.0	0.0906	0.0003	0.0014	83.41	70.89
15	4.451	224.133	63.898	421.662	210.626	5.787	5.574	197.549	83.573	18.782	300.0	0.0964	0.0003	0.0015	89.20	76.47
16	4.449	224.067	63.803	421.431	210.636	5.763	5.645	196.599	84.595	18.707	300.0	0.0994	0.0003	0.0015	94.96	82.11
17	4.447	224.000	63.706	421.200	210.642	5.739	5.717	195.641	85.625	18.635	300.0	0.0989	0.0003	0.0015	100.70	87.83
18	4.446	223.934	63.612	420.970	210.643	5.715	5.790	194.678	86.662	18.567	300.0	0.0928	0.0003	0.0015	106.41	93.62
19	4.444	223.868	63.514	420.741	210.639	5.690	5.863	193.706	87.708	18.502	300.0	0.0847	0.0003	0.0015	112.10	99.48
20	4.443	223.803	63.415	420.514	210.631	5.665	5.936	192.718	88.751	18.438	300.0	0.0928	0.0003	0.0015	117.77	105.42
21	4.441	223.737	63.315	420.287	210.617	5.640	6.010	191.722	89.802	18.378	300.0	0.0981	0.0003	0.0016	123.41	111.43
22	4.440	223.672	63.213	420.061	210.599	5.614	6.085	190.724	90.866	18.323	300.0	0.0864	0.0003	0.0016	129.02	117.51
23	4.438	223.608	63.111	419.837	210.575	5.588	6.161	189.709	91.929	18.270	300.0	0.0923	0.0003	0.0016	134.61	123.67
24	4.437	223.543	63.007	419.613	210.546	5.562	6.237	188.682	92.999	18.220	300.0	0.0986	0.0003	0.0016	140.17	129.91

COST
1957.078

First Corrected Values

G(1)0 = 241.651
G(2)0 = 67.995

NR	L	G1	G2	H1	H2	D1	D2	PH1	PH2	PT	PD	ERR	Z1	Z2	Q1	Q2
1	4.741	241.651	67.995	425.000	210.000	5.462	5.436	187.691	81.320	30.892	300.0	0.0479	0.0	0.0	5.43	5.44
2	4.740	241.583	67.897	424.781	210.002	5.456	5.511	186.650	82.401	30.842	300.0	0.0474	0.0003	0.0014	10.94	10.95
3	4.739	241.516	67.798	424.562	210.000	5.433	5.588	185.626	83.496	30.797	300.0	0.0815	0.0003	0.0015	16.37	16.53
4	4.736	241.449	67.697	424.345	209.992	5.403	5.664	184.566	84.589	30.753	300.0	0.0916	0.0003	0.0015	21.77	22.20
5	4.737	241.383	67.595	424.129	209.979	5.376	5.742	183.502	85.695	30.715	300.0	0.0889	0.0003	0.0015	27.15	27.94
6	4.736	241.317	67.492	423.914	209.960	5.347	5.821	182.419	86.805	30.680	300.0	0.0964	0.0003	0.0015	32.50	33.76
7	4.736	241.251	67.387	423.699	209.937	5.321	5.900	181.329	87.925	30.650	300.0	0.0935	0.0003	0.0016	37.82	39.66
8	4.735	241.186	67.281	423.487	209.908	5.293	5.981	180.231	89.062	30.625	300.0	0.0820	0.0003	0.0016	43.11	45.64
9	4.734	241.121	67.174	423.275	209.873	5.265	6.062	179.110	90.200	30.603	300.0	0.0859	0.0003	0.0016	48.37	51.70
10	4.734	241.056	67.065	423.064	209.835	5.236	6.145	177.979	91.350	30.587	300.0	0.0840	0.0003	0.0016	53.61	57.85
11	4.734	240.992	66.954	422.854	209.788	5.207	6.228	176.835	92.511	30.575	300.0	0.0791	0.0003	0.0016	58.82	64.08
12	4.734	240.928	66.842	422.646	209.737	5.177	6.312	175.663	93.675	30.567	300.0	0.0955	0.0003	0.0017	63.99	70.39
13	4.734	240.864	66.729	422.439	209.680	5.147	6.398	174.486	94.856	30.563	300.0	0.0913	0.0003	0.0017	69.14	76.79
14	4.734	240.801	66.614	422.233	209.617	5.116	6.485	173.294	96.047	30.566	300.0	0.0916	0.0003	0.0017	74.26	83.27
15	4.734	240.738	66.497	422.028	209.549	5.085	6.572	172.077	97.247	30.577	300.0	0.0994	0.0003	0.0018	79.34	89.84
16	4.734	240.676	66.379	421.824	209.475	5.053	6.662	170.852	98.463	30.592	300.0	0.0938	0.0003	0.0018	84.39	96.51
17	4.735	240.614	66.260	421.622	209.394	5.021	6.752	169.616	99.694	30.613	300.0	0.0769	0.0003	0.0018	89.42	103.26
18	4.735	240.552	66.138	421.421	209.308	4.987	6.844	168.349	100.934	30.640	300.0	0.0776	0.0003	0.0018	94.40	110.13
19	4.736	240.491	66.015	421.222	209.215	4.956	6.938	167.065	102.169	30.673	300.0	0.0732	0.0003	0.0019	99.36	117.06
20	4.737	240.430	65.890	421.023	209.118	4.922	7.033	165.760	103.456	30.713	300.0	0.0708	0.0003	0.0019	104.28	124.07
21	4.738	240.369	65.764	420.826	209.010	4.888	7.129	164.425	104.737	30.760	300.0	0.0781	0.0003	0.0019	109.17	131.20
22	4.740	240.309	65.636	420.631	208.898	4.853	7.227	163.079	106.038	30.815	300.0	0.0681	0.0002	0.0020	114.02	138.43
23	4.741	240.250	65.506	420.437	208.779	4.817	7.327	161.694	107.352	30.877	300.0	0.0776	0.0002	0.0020	118.84	145.76
24	4.743	240.190	65.374	420.244	208.654	4.780	7.428	160.284	108.683	30.947	300.0	0.0862	0.0002	0.0020	123.62	153.16

COST
3241.806

G(1)0 = 241.651 + 0.1
G(2)0 = 67.995

HR	L	G1	G2	H1	H2	D1	D2	PH1	PH2	PT	PO	ERR	Z1	Z2	Q1	Q2
1	4.743	241.751	67.995	425.000	210.000	5.473	5.454	187.367	81.566	30.953	300.0	0.0974	0.0	0.0	5.47	5.45
2	4.742	241.683	67.897	424.781	210.001	5.447	5.530	186.335	82.672	30.902	300.0	0.0911	0.0003	0.0014	10.92	10.9[illegible]
3	4.741	241.616	67.797	424.563	209.997	5.421	5.606	185.290	83.764	30.857	300.0	0.0901	0.0003	0.0015	16.34	16.59
4	4.740	241.549	67.695	424.346	209.987	5.394	5.684	184.230	84.861	30.815	300.0	0.0940	0.0003	0.0015	21.74	22.27
5	4.739	241.483	67.594	424.130	209.973	5.367	5.762	183.163	85.972	30.776	300.0	0.0872	0.0003	0.0015	27.10	28.04
6	4.738	241.417	67.493	423.915	209.955	5.339	5.841	182.079	87.086	30.745	300.0	0.0893	0.0003	0.0015	32.44	33.84
7	4.737	241.352	67.383	423.702	209.925	5.312	5.920	180.987	88.213	30.717	300.0	0.0837	0.0003	0.0016	37.75	39.30
8	4.737	241.286	67.279	423.489	209.896	5.283	6.001	179.874	89.344	30.692	300.0	0.0906	0.0003	0.0016	43.04	45.30
9	4.736	241.221	67.171	423.278	209.862	5.255	6.083	178.751	90.457	30.672	300.0	0.0893	0.0003	0.0016	48.29	51.49
10	4.736	241.157	67.061	423.067	209.820	5.226	6.165	177.516	91.641	30.657	300.0	0.0869	0.0003	0.0016	53.52	58.05
11	4.736	241.093	66.950	422.856	209.773	5.197	6.249	176.408	92.806	30.647	300.0	0.0883	0.0003	0.0017	58.71	64.37
12	4.735	241.029	66.838	422.650	209.721	5.166	6.334	175.292	93.975	30.640	300.0	0.0954	0.0003	0.0017	63.88	70.53
13	4.735	240.965	66.724	422.443	209.662	5.136	6.420	174.111	95.157	30.640	300.0	0.0920	0.0003	0.0017	69.02	77.05
14	4.735	240.902	66.609	422.238	209.598	5.105	6.507	172.910	96.352	30.645	300.0	0.0940	0.0003	0.0017	74.12	83.56
15	4.736	240.840	66.492	422.033	209.528	5.075	6.595	171.705	97.562	30.657	300.0	0.0785	0.0003	0.0018	79.20	90.15
16	4.736	240.777	66.373	421.832	209.452	5.045	6.685	170.471	98.782	30.673	300.0	0.0745	0.0003	0.0018	84.24	96.84
17	4.737	240.715	66.253	421.628	209.370	5.015	6.776	169.209	100.007	30.695	300.0	0.0874	0.0003	0.0018	89.25	103.51
18	4.737	240.654	66.131	421.428	209.282	4.977	6.868	167.929	101.251	30.723	300.0	0.0764	0.0003	0.0018	94.23	110.43
19	4.738	240.592	66.005	421.229	209.187	4.945	6.962	166.655	102.515	30.760	300.0	0.0720	0.0003	0.0019	99.17	117.44
20	4.739	240.532	65.882	421.031	209.085	4.915	7.057	165.331	103.786	30.802	300.0	0.0815	0.0003	0.0019	104.08	124.50
21	4.740	240.471	65.755	420.834	208.977	4.875	7.154	163.961	105.057	30.850	300.0	0.0796	0.0003	0.0019	108.96	131.05
22	4.742	240.411	65.627	420.639	208.865	4.841	7.253	162.542	106.379	30.905	300.0	0.0710	0.0002	0.0020	113.80	138.91
23	4.743	240.352	65.495	420.445	208.744	4.805	7.354	161.258	107.702	30.973	300.0	0.0671	0.0002	0.0020	118.60	145.76
24	4.745	240.293	65.364	420.253	208.617	4.768	7.456	159.549	109.042	31.047	300.0	0.0632	0.0002	0.0020	123.37	153.70

COST
3250.421

G(1)0 = 241.651 + 0.1
G(2)0 = 67.995 + 0.1

HR	L	G1	G2	H1	H2	U1	U2	PH1	PH2	PT	PD	ERR	Z1	Z2	Q1	Q2
1	4.744	241.751	68.095	425.000	210.000	5.501	5.301	188.374	80.334	31.010	300.0	0.0825	0.0	0.0	5.50	5.35
2	4.743	241.683	67.995	424.780	210.000	5.475	5.455	187.348	81.607	30.957	300.0	0.0881	0.0003	0.0014	10.98	10.84
3	4.742	241.615	67.900	424.551	210.007	5.449	5.531	186.308	82.680	30.907	300.0	0.0995	0.0003	0.0014	16.42	16.37
4	4.741	241.548	67.800	424.343	210.005	5.422	5.607	185.265	83.778	30.862	300.0	0.0955	0.0003	0.0015	21.85	21.77
5	4.740	241.482	67.699	424.126	209.994	5.395	5.684	184.206	84.870	30.820	300.0	0.0980	0.0003	0.0015	27.24	27.44
6	4.739	241.416	67.597	423.910	209.979	5.368	5.763	183.141	85.950	30.733	300.0	0.0901	0.0003	0.0015	32.61	33.42
7	4.738	241.349	67.495	423.695	209.959	5.341	5.841	182.061	87.102	30.750	300.0	0.0861	0.0003	0.0015	37.95	39.25
8	4.737	241.284	67.388	423.481	209.934	5.315	5.921	180.969	88.229	30.722	300.0	0.0826	0.0003	0.0015	43.26	45.13
9	4.737	241.218	67.281	423.269	209.904	5.285	6.002	179.860	89.360	30.697	300.0	0.0833	0.0003	0.0016	48.55	51.19
10	4.736	241.153	67.174	423.057	209.865	5.257	6.084	178.740	90.504	30.677	300.0	0.0801	0.0003	0.0016	53.81	57.27
11	4.736	241.089	67.064	422.847	209.817	5.227	6.166	177.595	91.651	30.660	300.0	0.0962	0.0003	0.0016	59.03	63.44
12	4.736	241.025	66.955	422.637	209.763	5.196	6.250	176.448	92.815	30.650	300.0	0.0874	0.0003	0.0017	64.23	69.69
13	4.735	240.961	66.841	422.429	209.727	5.164	6.335	175.288	93.991	30.645	300.0	0.0767	0.0003	0.0017	69.40	76.02
14	4.735	240.897	66.727	422.225	209.665	5.135	6.420	174.095	95.166	30.643	300.0	0.0938	0.0003	0.0017	74.54	82.44
15	4.736	240.834	66.612	422.017	209.605	5.107	6.507	172.899	96.363	30.648	300.0	0.0901	0.0003	0.0017	79.64	88.95
16	4.736	240.772	66.495	421.815	209.555	5.076	6.596	171.678	97.567	30.658	300.0	0.0972	0.0003	0.0018	84.72	95.54
17	4.736	240.709	66.379	421.610	209.457	5.044	6.685	170.450	98.785	30.675	300.0	0.0891	0.0003	0.0018	89.76	102.23
18	4.737	240.647	66.256	421.408	209.376	5.012	6.776	169.189	100.014	30.697	300.0	0.0999	0.0003	0.0018	94.78	109.01
19	4.737	240.585	66.134	421.207	209.283	4.979	6.869	167.935	101.264	30.727	300.0	0.0745	0.0003	0.0018	99.76	115.87
20	4.738	240.524	66.010	421.006	209.194	4.946	6.962	166.642	102.521	30.762	300.0	0.0757	0.0003	0.0019	104.70	122.84
21	4.739	240.464	65.885	420.810	209.093	4.912	7.058	165.324	103.792	30.803	300.0	0.0803	0.0003	0.0019	109.61	129.87
22	4.740	240.403	65.758	420.615	208.980	4.877	7.154	163.978	105.077	30.852	300.0	0.0938	0.0003	0.0019	114.49	137.05
23	4.742	240.343	65.627	420.415	208.872	4.842	7.253	162.617	106.380	30.905	300.0	0.0945	0.0002	0.0020	119.33	144.30
24	4.743	240.284	65.499	420.224	208.751	4.806	7.353	161.245	107.705	30.973	300.0	0.0813	0.0002	0.0020	124.14	151.65

CCSF
3250.618

Second Corrected Values

G(1)0 = 241.259
G(2)0 = 68.048

NR	L	G1	G2	H1	H2	D1	D2	PH1	PH2	PT	PD	ERR	Z1	Z2	Q1	Q2
1	4.737	241.259	68.048	425.000	210.000	5.531	5.325	189.482	79.735	30.700	300.0	0.0825	0.0	0.0	5.53	5.33
2	4.735	241.191	67.952	424.779	210.010	5.505	5.399	188.459	80.603	30.643	300.0	0.0852	0.0003	0.0014	11.04	10.72
3	4.734	241.123	67.855	424.556	210.015	5.480	5.474	187.445	81.677	30.590	300.0	0.0872	0.0003	0.0014	16.52	16.20
4	4.733	241.056	67.757	424.339	210.016	5.453	5.549	186.409	82.955	30.540	300.0	0.0952	0.0003	0.0015	21.97	21.75
5	4.732	240.989	67.657	424.121	210.011	5.427	5.626	185.367	84.043	30.495	300.0	0.0898	0.0003	0.0015	27.40	27.37
6	4.731	240.923	67.555	423.904	210.001	5.400	5.703	184.311	85.146	30.455	300.0	0.0903	0.0003	0.0015	32.80	33.05
7	4.730	240.858	67.453	423.688	209.985	5.373	5.781	183.238	86.249	30.415	300.0	0.0981	0.0003	0.0015	38.17	38.86
8	4.729	240.790	67.350	423.472	209.965	5.346	5.860	182.159	87.365	30.382	300.0	0.0842	0.0003	0.0015	43.52	44.72
9	4.728	240.725	67.244	423.259	209.940	5.318	5.940	181.073	88.493	30.353	300.0	0.0815	0.0003	0.0015	48.85	50.66
10	4.728	240.659	67.138	423.046	209.909	5.290	6.020	179.967	89.624	30.328	300.0	0.0813	0.0003	0.0016	54.12	56.64
11	4.727	240.595	67.030	422.834	209.872	5.261	6.102	178.830	90.761	30.307	300.0	0.0967	0.0003	0.0016	59.39	62.74
12	4.727	240.530	66.920	422.624	209.830	5.232	6.185	177.708	91.915	30.292	300.0	0.0857	0.0003	0.0016	64.62	68.96
13	4.727	240.466	66.807	422.414	209.785	5.203	6.268	176.554	93.073	30.280	300.0	0.0930	0.0003	0.0017	69.82	75.23
14	4.727	240.402	66.697	422.206	209.727	5.173	6.353	175.398	94.249	30.275	300.0	0.0766	0.0003	0.0017	74.99	81.59
15	4.727	240.339	66.583	421.999	209.673	5.143	6.439	174.211	95.427	30.273	300.0	0.0884	0.0003	0.0017	80.14	88.02
16	4.727	240.276	66.467	421.793	209.605	5.112	6.526	173.019	96.522	30.276	300.0	0.0803	0.0003	0.0017	85.25	94.55
17	4.727	240.213	66.350	421.588	209.535	5.081	6.614	171.807	97.826	30.288	300.0	0.0796	0.0003	0.0018	90.33	101.16
18	4.727	240.151	66.231	421.385	209.458	5.049	6.703	170.566	99.040	30.303	300.0	0.0908	0.0003	0.0018	95.38	107.87
19	4.728	240.089	66.111	421.183	209.375	5.017	6.794	169.317	100.268	30.325	300.0	0.0903	0.0003	0.0018	100.40	114.66
20	4.728	240.027	65.989	420.982	209.287	4.985	6.885	168.051	101.511	30.353	300.0	0.0850	0.0003	0.0019	105.38	121.55
21	4.729	239.966	65.865	420.783	209.191	4.952	6.980	166.769	102.769	30.388	300.0	0.0745	0.0003	0.0019	110.33	128.53
22	4.730	239.905	65.740	420.584	209.090	4.918	7.076	165.459	104.040	30.430	300.0	0.0706	0.0003	0.0019	115.25	135.60
23	4.731	239.845	65.613	420.388	208.982	4.883	7.172	164.129	105.325	30.478	300.0	0.0670	0.0003	0.0019	120.13	142.76
24	4.733	239.785	65.484	420.192	208.868	4.845	7.271	162.755	106.623	30.533	300.0	0.0654	0.0002	0.0020	124.98	150.05

COST
3259.054

G(1)0 = 241.259 + 0.1
G(2)0 = 68.048

HR	L	G1	G2	H1	H2	D1	D2	PH1	PH2	PT	PD	ERR	Z1	Z2	Q1	Q2
1	4.738	241.359	68.048	425.000	210.000	5.522	5.343	189.162	79.995	30.757	300.0	0.0857	0.0	0.0	5.52	5.34
2	4.737	241.291	67.952	424.779	210.009	5.497	5.417	188.141	81.060	30.700	300.0	0.0991	0.0003	0.0014	11.02	10.76
3	4.736	241.223	67.854	424.559	210.013	5.471	5.492	187.117	82.136	30.646	300.0	0.0474	0.0003	0.0014	16.49	16.25
4	4.734	241.156	67.756	424.340	210.012	5.445	5.569	186.088	83.229	30.602	300.0	0.0911	0.0003	0.0015	21.93	21.87
5	4.733	241.089	67.656	424.122	210.006	5.418	5.645	185.055	84.320	30.557	300.0	0.0886	0.0003	0.0015	27.35	27.47
6	4.732	241.023	67.554	423.906	209.994	5.391	5.723	183.979	85.422	30.517	300.0	0.0825	0.0003	0.0015	32.74	33.19
7	4.732	240.957	67.451	423.690	209.976	5.364	5.801	182.904	86.530	30.480	300.0	0.0867	0.0003	0.0015	38.11	38.99
8	4.731	240.891	67.347	423.475	209.956	5.336	5.880	181.812	87.642	30.447	300.0	0.0994	0.0003	0.0015	43.44	44.87
9	4.730	240.825	67.241	423.261	209.929	5.309	5.960	180.723	88.774	30.420	300.0	0.0930	0.0003	0.0015	48.75	50.33
10	4.730	240.760	67.134	423.049	209.896	5.280	6.041	179.614	89.910	30.397	300.0	0.0796	0.0003	0.0016	54.03	55.87
11	4.729	240.696	67.026	422.838	209.858	5.252	6.123	178.482	91.050	30.377	300.0	0.0925	0.0003	0.0015	59.28	62.39
12	4.729	240.631	66.916	422.627	209.814	5.222	6.205	177.338	92.201	30.362	300.0	0.0946	0.0003	0.0016	64.51	68.90
13	4.728	240.567	66.805	422.418	209.765	5.193	6.290	176.193	93.367	30.353	300.0	0.0847	0.0003	0.0017	69.70	75.49
14	4.728	240.503	66.692	422.211	209.710	5.163	6.374	175.017	94.542	30.348	300.0	0.0835	0.0003	0.0017	74.86	81.86
15	4.728	240.440	66.577	422.004	209.650	5.133	6.461	173.841	95.731	30.350	300.0	0.0781	0.0003	0.0017	80.00	88.32
16	4.729	240.377	66.461	421.799	209.583	5.102	6.548	172.627	96.922	30.355	300.0	0.0964	0.0003	0.0017	85.10	94.87
17	4.729	240.315	66.344	421.595	209.511	5.070	6.637	171.407	98.129	30.367	300.0	0.0979	0.0003	0.0018	90.17	101.51
18	4.729	240.252	66.225	421.392	209.433	5.039	6.727	170.177	99.353	30.385	300.0	0.0859	0.0003	0.0018	95.21	108.23
19	4.730	240.191	66.104	421.190	209.348	5.006	6.818	168.916	100.584	30.408	300.0	0.0920	0.0003	0.0018	100.21	115.05
20	4.731	240.129	65.981	420.990	209.255	4.973	6.910	167.640	101.831	30.436	300.0	0.0905	0.0003	0.0019	105.19	121.96
21	4.731	240.068	65.857	420.791	209.161	4.940	7.005	166.343	103.091	30.475	300.0	0.0911	0.0003	0.0019	110.13	128.97
22	4.732	240.007	65.731	420.595	209.058	4.906	7.100	165.021	104.365	30.518	300.0	0.0957	0.0003	0.0019	115.03	136.07
23	4.734	239.947	65.604	420.397	208.946	4.871	7.196	163.694	105.659	30.570	300.0	0.0771	0.0003	0.0019	119.90	143.25
24	4.735	239.887	65.474	420.202	208.852	4.836	7.297	162.328	106.966	30.626	300.0	0.0781	0.0002	0.0020	124.74	150.56

COST
3217.347

$G(1)0 = 241.259 + 0.1$
$G(2)0 = 68.048 + 0.1$

HR	C	G1	G2	H1	H2	D1	D2	PH1	PH2	PT	PD	ERR	Z1	Z2	Q1	Q2
1	4.740	241.359	68.148	425.000	210.000	5.549	5.271	190.134	78.948	30.818	300.0	0.0999	0.0	0.0	5.55	5.27
2	4.738	241.291	68.053	424.778	210.014	5.524	5.344	189.133	80.015	30.760	300.0	0.0923	0.0003	0.0014	11.07	10.81
3	4.737	241.225	67.957	424.557	210.023	5.499	5.419	188.124	81.087	30.705	300.0	0.0842	0.0003	0.0014	16.57	16.03
4	4.736	241.155	67.859	424.337	210.027	5.472	5.493	187.091	82.158	30.652	300.0	0.0989	0.0003	0.0014	22.04	21.55
5	4.735	241.088	67.761	424.118	210.028	5.446	5.570	186.064	83.250	30.605	300.0	0.0803	0.0003	0.0015	27.49	27.10
6	4.733	241.021	67.661	423.900	210.020	5.420	5.646	185.014	84.341	30.560	300.0	0.0852	0.0003	0.0015	32.91	32.74
7	4.732	240.955	67.559	423.683	210.008	5.393	5.723	183.949	85.438	30.518	300.0	0.0972	0.0003	0.0015	38.30	38.47
8	4.732	240.888	67.455	423.467	209.992	5.365	5.801	182.876	86.544	30.482	300.0	0.0989	0.0003	0.0015	43.67	44.27
9	4.731	240.822	67.352	423.252	209.970	5.338	5.881	181.798	87.664	30.450	300.0	0.0879	0.0003	0.0015	49.00	50.15
10	4.730	240.757	67.246	423.039	209.945	5.310	5.960	180.699	88.788	30.422	300.0	0.0913	0.0003	0.0016	54.31	56.11
11	4.730	240.692	67.139	422.826	209.910	5.282	6.042	179.592	89.925	30.398	300.0	0.0845	0.0003	0.0016	59.60	62.15
12	4.729	240.627	67.031	422.615	209.872	5.253	6.123	178.462	91.065	30.378	300.0	0.0947	0.0003	0.0016	64.85	68.27
13	4.729	240.563	66.921	422.405	209.829	5.224	6.206	177.320	92.217	30.363	300.0	0.0986	0.0003	0.0016	70.07	74.48
14	4.729	240.499	66.809	422.196	209.780	5.195	6.290	176.177	93.387	30.355	300.0	0.0815	0.0003	0.0017	75.27	80.77
15	4.728	240.435	66.697	421.988	209.725	5.165	6.375	175.005	94.559	30.350	300.0	0.0859	0.0003	0.0017	80.43	87.14
16	4.728	240.371	66.582	421.781	209.664	5.134	6.461	173.817	95.741	30.350	300.0	0.0923	0.0003	0.0017	85.57	93.61
17	4.729	240.309	66.466	421.575	209.598	5.104	6.549	172.620	96.940	30.357	300.0	0.0837	0.0003	0.0017	90.67	100.15
18	4.729	240.248	66.349	421.371	209.526	5.072	6.637	171.403	98.148	30.368	300.0	0.0803	0.0003	0.0018	95.74	106.79
19	4.729	240.184	66.229	421.168	209.447	5.040	6.727	170.160	99.365	30.385	300.0	0.0906	0.0003	0.0018	100.78	113.52
20	4.730	240.122	66.105	420.967	209.363	5.008	6.818	168.905	100.595	30.408	300.0	0.0928	0.0003	0.0018	105.79	120.34
21	4.731	240.060	65.980	420.768	209.272	4.975	6.911	167.630	101.843	30.438	300.0	0.0884	0.0003	0.0019	110.77	127.25
22	4.731	239.999	65.852	420.567	209.170	4.942	7.005	166.337	103.105	30.475	300.0	0.0835	0.0003	0.0019	115.71	134.25
23	4.732	239.939	65.730	420.369	209.072	4.908	7.101	165.025	104.379	30.515	300.0	0.0803	0.0003	0.0019	120.62	141.35
24	4.734	239.878	65.609	420.173	208.963	4.873	7.198	163.675	105.667	30.568	300.0	0.0903	0.0003	0.0019	125.49	143.55

COST
3218.421

Third Corrected Values

G(1)0 = 241.258
G(2)0 = 68.050

HR	L	G1	G2	H1	H2	D1	D2	PH1	PH2	PT	PD	ERR	Z1	Z2	Q1	Q2
1	4.737	241.258	68.050	425.000	210.000	5.532	5.325	189.501	79.703	30.700	300.0	0.0916	0.0	0.0	5.53	5.32
2	4.735	241.190	67.954	424.779	210.010	5.505	5.377	188.458	80.776	30.643	300.0	0.0930	0.0003	0.0014	11.04	10.72
3	4.734	241.122	67.857	424.558	210.010	5.480	5.472	187.400	81.849	30.590	300.0	0.0952	0.0003	0.0014	16.52	16.10
4	4.733	241.055	67.759	424.339	210.010	5.454	5.548	186.440	82.936	30.542	300.0	0.0825	0.0003	0.0015	21.97	21.74
5	4.732	240.988	67.659	424.121	210.011	5.428	5.624	185.388	84.021	30.495	300.0	0.0957	0.0003	0.0015	27.40	27.37
6	4.731	240.921	67.558	423.903	210.002	5.401	5.701	184.334	85.119	30.453	300.0	0.0933	0.0003	0.0015	32.80	33.07
7	4.730	240.855	67.456	423.687	209.987	5.374	5.779	183.271	86.231	30.417	300.0	0.0815	0.0003	0.0015	38.17	38.65
8	4.729	240.789	67.352	423.472	209.966	5.346	5.858	182.183	87.339	30.382	300.0	0.0954	0.0003	0.0015	43.52	44.72
9	4.728	240.724	67.247	423.258	209.941	5.319	5.938	181.097	88.467	30.353	300.0	0.0828	0.0003	0.0015	48.84	50.64
10	4.728	240.659	67.140	423.045	209.910	5.291	6.019	179.992	89.599	30.328	300.0	0.0813	0.0003	0.0015	54.13	56.66
11	4.727	240.594	67.032	422.834	209.875	5.262	6.100	178.865	90.735	30.307	300.0	0.0933	0.0003	0.0016	59.39	62.75
12	4.727	240.529	66.923	422.625	209.831	5.233	6.183	177.737	91.890	30.292	300.0	0.0813	0.0003	0.0016	[illegible]	68.94
13	4.727	240.465	66.812	422.414	209.784	5.204	6.265	176.585	93.048	30.280	300.0	0.0572	0.0003	0.0017	69.83	75.21
14	4.727	240.401	66.699	422.205	209.731	5.174	6.351	175.410	94.217	30.273	300.0	0.0934	0.0003	0.0017	75.00	81.55
15	4.727	240.338	66.585	421.998	209.672	5.144	6.437	174.246	95.403	30.273	300.0	0.0731	0.0003	0.0017	80.15	[illegible]
16	4.727	240.275	66.470	421.792	209.607	5.113	6.524	173.041	96.591	30.277	300.0	0.0920	0.0003	0.0017	85.26	94.52
17	4.727	240.212	66.355	421.588	209.537	5.082	6.612	171.829	97.795	30.237	300.0	0.0871	0.0003	0.0018	90.34	101.12
18	4.727	240.150	66.234	421.385	209.460	5.050	6.701	170.591	99.009	30.302	300.0	0.0954	0.0003	0.0018	95.39	107.83
19	4.726	240.088	66.114	421.182	209.378	5.018	6.792	169.342	100.257	30.323	300.0	0.0974	0.0003	0.0018	100.41	114.62
20	4.726	240.025	65.992	420.981	209.285	4.985	6.884	168.077	101.481	30.352	300.0	0.0905	0.0003	0.0018	105.40	121.51
21	4.729	239.965	65.868	420.782	209.194	4.952	6.976	166.794	102.739	30.367	300.0	0.0801	0.0003	0.0019	110.35	128.49
22	4.730	239.904	65.743	420.564	209.093	4.915	7.073	165.470	104.004	30.427	300.0	0.0994	0.0003	0.0019	115.27	135.56
23	4.731	239.844	65.618	420.367	208.985	4.884	7.170	164.140	105.280	30.475	300.0	0.0452	0.0003	0.0019	120.15	142.73
24	4.733	239.784	65.487	420.171	208.871	4.849	7.268	162.794	106.594	30.532	300.0	0.0806	0.0002	0.0020	125.00	[illegible]